



NEHRU COLLEGE OF ENGINEERING AND RESEARCH CENTRE
(NAAC Accredited)
(Approved by AICTE, Affiliated to APJ Abdul Kalam Technological University,
Kerala)



DEPARTMENT OF MECHANICAL ENGINEERING

COURSE MATERIALS



ME308 COMPUTER AIDED DESIGN AND ANALYSIS

VISION OF THE INSTITUTION

To mould true citizens who are millennium leaders and catalysts of change through excellence in education.

MISSION OF THE INSTITUTION

NCERC is committed to transform itself into a center of excellence in Learning and Research in Engineering and Frontier Technology and to impart quality education to mould technically competent citizens with moral integrity, social commitment and ethical values.

We intend to facilitate our students to assimilate the latest technological know-how and to imbibe discipline, culture and spiritually, and to mould them in to technological giants, dedicated research scientists and intellectual leaders of the country who can spread the beams of light and happiness among the poor and the underprivileged.

ABOUT DEPARTMENT

- ◆ Established in: 2002
- ◆ Course offered : B.Tech in Mechanical Engineering
- ◆ Approved by AICTE New Delhi and Accredited by NAAC

- ◆ Affiliated to the University of Dr. A P J Abdul Kalam Technological University.

DEPARTMENT VISION

Producing internationally competitive Mechanical Engineers with social responsibility & sustainable employability through viable strategies as well as competent exposure oriented quality education.

DEPARTMENT MISSION

1. Imparting high impacted education by providing conducive teaching learning environment.
2. Fostering effective modes of continuous learning process with moral & ethical values.
3. Enhancing leadership qualities with social commitment, professional attitude, unity, team spirit & communication skill.
4. Introducing the present scenario in research & development through collaborative efforts blended with industry & institution.

PROGRAMME EDUCATIONAL OBJECTIVES

PEO1: Graduates shall have strong practical & technical exposures in the field of Mechanical Engineering & will contribute to the society through innovation & enterprise.

PEO2: Graduates will have the demonstrated ability to analyze, formulate & solve design engineering / thermal engineering / materials & manufacturing / design issues & real life problems.

PEO3: Graduates will be capable of pursuing Mechanical Engineering profession with good communication skills, leadership qualities, team spirit & communication skills.

PEO4: Graduates will sustain an appetite for continuous learning by pursuing higher education & research in the allied areas of technology.

PROGRAM OUTCOMES (POS)

Engineering Graduates will be able to:

1. **Engineering knowledge:** Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.
2. **Problem analysis:** Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.
3. **Design/development of solutions:** Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.

4. **Conduct investigations of complex problems:** Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.
5. **Modern tool usage:** Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modeling to complex engineering activities with an understanding of the limitations.
6. **The engineer and society:** Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.
7. **Environment and sustainability:** Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.
8. **Ethics:** Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.
9. **Individual and teamwork:** Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.
10. **Communication:** Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.
11. **Project management and finance:** Demonstrate knowledge and understanding of the engineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.
12. **Life-long learning:** Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

PROGRAM SPECIFIC OUTCOMES (PSO)

PSO1: graduates able to apply principles of engineering, basic sciences & analytics including multi variant calculus & higher order partial differential equations..

PSO2: Graduates able to perform modeling, analyzing, designing & simulating physical systems, components & processes.

PSO3: Graduates able to work professionally on mechanical systems, thermal systems & production systems.

COURSE OUTCOMES

CO1	Understand the basics of Computer Aided Design methods and procedures.
CO2	Apply various transformation techniques involved in Computer Aided Design
CO3	Prepare the representation and modeling of curves and surfaces
CO4	Apply the concepts of Solid Modeling
CO5	Analyze the basic concepts of Finite Element analysis and procedures and to solve simple structural problems
CO6	Evaluate the various interpolation functions and iso-parametric formulation

MAPPING OF COURSE OUTCOMES WITH PROGRAM OUTCOMES

	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12	PSO1	PSO2	PSO3
CO1	2				3							3	2	3	-
CO2	3		2		3							3	2	2	-
CO3	3		2		3							3	2	3	-
CO4	3		2		3							3	1	3	-
CO5	3	2										3	3	3	-
CO6	3	2										3	3	2	-

Note: H-Highly correlated=3, M-Medium correlated=2, L-Less correlated=1

SYLLABUS

Course code	Course Name	L-T-P-Credits	Year of Introduction
ME308	COMPUTER AIDED DESIGN AND ANALYSIS	3-0-0-3	2016
Prerequisite: ME201 Mechanics of solids			
Course Objectives:			
<ol style="list-style-type: none"> 1. To impart basic knowledge on Computer Aided Design methods and procedures 2. To introduce the fundamentals of solid modelling 3. To introduce the concepts of finite element analysis procedures. 			
Syllabus			
Introduction to CAD/CAM, Basics of geometric and solid modeling, transformation, representation points, lines, surfaces and solid models. Introduction to finite element analysis, solution procedures, interpolation, isoparametric formulation, applications.			
Expected outcome:			
The students will be able to			
<ol style="list-style-type: none"> 1. Gain a basic knowledge on Computer Aided Design methods and procedures 2. Understand the fundamentals of solid modelling 3. Have a basic knowledge in finite element analysis procedures. 			
Text Books:			
<ol style="list-style-type: none"> 1. M.P. Groover, E.M. Zimmers, Jr. CAD/CAM; Computer Aided Design and Manufacturing, Prentice Hall of India, 1987 2. T. R. Chandrupatla and A. D. Belagundu, Introduction to Finite Elements in Engineering, Pearson Education, 2001 			
References:			
<ol style="list-style-type: none"> 1. Chris McMahon and Jimmie Browne - CAD/CAM – Principle Practice and Manufacturing Management, Addison Wesley England, 1998 2. D. F. Rogers and J. A. Adams, Mathematical Elements in Computer Graphics, McGraw-Hill, 1990 3. Daryl Logan, A First course in Finite Element Method, Thomson Learning, 2007 4. David V Hutton, Fundamentals of Finite Element Analysis, THM, 2003 5. Donald Hearn, M. Pauline Baker and Warren Carithers, Computer Graphics with open GL, Pearson Education, 2001 6. Grigore Burdea, Philippe Coiffet, Virtual Reality Technology, John Wiley and sons, 2003 7. Ibrahim Zeid, CAD/ CAM Theory and Practice, McGraw Hill, 2007 8. P. Radhakrishnan and S. Subramanyan, CAD /CAM / CIM, New Age Int. Ltd., 2008 			

Course Plan			
Module	Contents	Hours	End Sem. Exam Marks
I	Introduction to CAD, Historical developments, Industrial look at CAD, Comparison of CAD with traditional designing, Application of computers in Design	2	15%
	Basics of geometric and solid modeling, Packages for CAD/CAM/CAE/CAPP	1	
	Hardware in CAD components, user interaction devices, design database, graphic Standards, data Exchange Formats, virtual Reality.	4	
II	Transformation of points and line, 2-D rotation, reflection, scaling and combined transformation, homogeneous coordinates, 3-D scaling.	4	15%
	Shearing, rotation, reflection and translation, combined transformations, orthographic and perspective projections, reconstruction of 3-D objects.	3	
FIRST INTERNAL EXAM			
III	Algebraic and geometric forms, tangents and normal, blending functions, reparametrization, straight lines, conics, cubic splines, Bezier curves and B-spline curves.	4	15%
	Plane surface, ruled surface, surface of revolution, tabulated cylinder, bi-cubic surface, bezier surface, B-spline surfaces and their modeling techniques.	3	
IV	Solid models and representation scheme, boundary representation, constructive solid geometry.	3	15%
	Sweep representation, cell decomposition, spatial occupancy enumeration, coordinate systems for solid modeling.	4	
SECOND INTERNAL EXAM			
V	Introduction to finite element analysis - steps involved in FEM- Preprocessing phase - discretisation - types of elements	2	20%
	Formulation of stiffness matrix (direct method, 1-D element) - formulation of load vector - assembly of global equations - implementation of boundary conditions - solution procedure - post processing phase	3	
	Simple problems with axial bar element (structural problems only)	2	
VI	Interpolation - selection of interpolation functions - CST element - isoparametric formulation (using minimum PE theorem) - Gauss-quadrature	4	20%

	Solution of 2D plane stress solid mechanics problems (linear static analysis)	3	
END SEMESTER EXAM			

Question Paper Pattern

Maximum marks: 100

Time: 3 hrs

The question paper should consist of three parts

Part A

There should be 2 questions each from module I and II

Each question carries 10 marks

Students will have to answer any three questions out of 4 (3X10 marks =30 marks)

Part B

There should be 2 questions each from module III and IV

Each question carries 10 marks

Students will have to answer any three questions out of 4 (3X10 marks =30 marks)

Part C

There should be 3 questions each from module V and VI

Each question carries 10 marks

Students will have to answer any four questions out of 6 (4X10 marks =40 marks)

Note: Each question can have a maximum of four sub questions, if needed.

QUESTION BANK**MODULE I**

Q:NO:	QUESTIONS	CO	KL
1	Describe about components in design database	CO1	K2
2	Write down the significance of various data exchange formats and their applications in CAD.	CO1	K3
3	Explain Graphic standards	CO1	K2
4	Distinguish between traditional drawing methods to CAD.	CO1	K4
5	Write down the advantage of Computer graphics packages .	CO1	K2
6	What you mean by Virtual reality.	CO1	K2
7	What you mean by Graphics kernal system.	CO1	K2
8	List out CAD, CAM and CAA softwares.	CO1	K4
9	Describe the steps involved in design process.	CO1	K2
10	Explain graphic input devices used in CAD	CO1	K2
11	Discuss the benefits using CAD.	CO1	K2

MODULE II

1	Explain geometric transformation.	CO2	K2
2	Differentiate Translation and Scaling.	CO2	K4
3	What is the need for need for parametric transformation.	CO2	K2
4	Explain Rotaional Transformation in 3D	CO2	K5
5	Translate the square ABCD having the coordinates A(0,0),B(3,0),C(3,3) and D(0,3) by 2units along both direction and then scale it by 1.5units in X direction and 0.5 units along Y direction. Plot the sketches.	CO2	K5
6	Explain Perspective Projection and Orthographic projection in matri form	CO2	K3
7	Rotate the rectangle given as (0,0),(2,0),(2,2),(0,2) 30 deg CCW about its centroid and find the new coordinates of rectangle.	CO2	K5
8	With neat sketches explain in detail the windowing and clipping processes	CO2	K4
9	Differentiate wireframe modeling and solid modeling with neat sketches	CO2	K2

MODULE III

1	What you mean by DDA Algorithm?	CO3	K3
2	Explain the procedure of Bresenham's Algorithm.	CO3	K3
3	Write a short note on Syanthetic curves	CO3	K2
4	Explain different types of Analytical curves.	CO3	K5
5	Derive the equation for Cubic spline curve	CO3	K5
6	Derive the equation for B-spline curve	CO3	K5
7	Derive the equation for Bezier curve	CO3	K2
8	Differentiate Hermite cubic spline and Bezier curve	CO3	K2
9	Explain various surfaces used in CAD Modelling.	CO3	K5
10	Explain Bezier surface with neat sketch	CO3	K2

MODULE IV

1	Discuss the features of solid modeling packages	CO4	K2
2	Point out the advantages and disadvantages of B-rep method.	CO4	K4
3	Illustrate constructive solid geometry method in detail	CO4	K4
4	Write a short note on Solid modelling techniques.	CO4	K3
5	Explain cell decomposition and spatial occupancy enumeration	CO4	K1
6	Compare cell decomposition and primitive instancing	CO4	K4
7	Write a short note of sweeping methods.	CO4	K3
8	Explain spatial occupancy and enumeration technique	CO4	K2

MODULE V

1	Explain the term preprocessing in relation with FEA.	CO5	K2
2	Justify Displacement functions are selected as polynomial function.	CO5	K4
3	Discuss factors to be considered during discretization process of FEA.	CO5	K4
4	Derive the expression for load or force vector in FEA	CO5	K4
5	Derive the expression of stiffness matrix of 1D bar element	CO5	K4
6	Write applications of FEA	CO5	K3
7	Identify different types of finite elements and their applications	CO5	K2
8	Explain the assembling procedure for forming finite element equation for a 1 D Bar element	CO5	K4
9	Discuss factors which are considered during discretization process of FEA	CO5	K2

MODULE VI

1	Derive the displacement function for 1 D element..	CO6	K4
2	Justify polynomial functions are generally used as shape function	CO6	K5
3	Differentiate isoparametric, sub parametric and super parametric elements	CO6	K3
4	Derive the shape function for constant strain triangular element.	CO6	K4
5	Differentiate plane stress and plane strain problems	CO6	K3
6	Explain Gaussian quadrature in the analysis of plane stress problem.	CO6	K2
7	Derive the shape function for 1 D element.	CO6	K4

APPENDIX 1

CONTENT BEYOND THE SYLLABUS

S:NO.	WEB SOURCE REFERENCES
1	http://www.in.zapmetasearch.co
2	http://www.knowledge.autodesk.com
3	http://www.docs.heagonmi.com

Module1

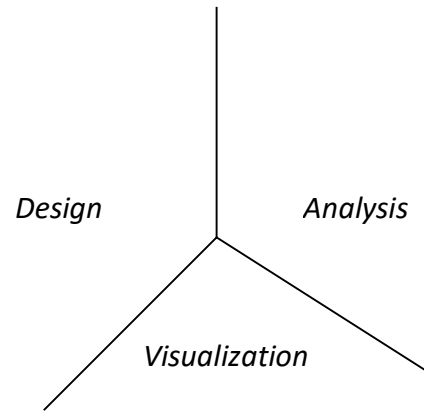
Syllabus: Introduction to CAD , Historical developments, Industrial look at CAD, Comparison of CAD with traditional designing, Application of computers in Design Basics of geometric and solid modeling, Packages for CAD/CAM/CAE/CAPP Hardware in CAD components, user interaction devices, design database, graphic Standards, data Exchange Formats, virtual Reality..

MODULE I

Introduction

In general, a Computer Aided Design (CAD) package has three components: a) Design, b) Analysis, and c) Visualization, as shown in the sketch. A brief description of these components follows.

- a) **Design:** Design refers to geometric modeling, i.e., 2-D and 3-D modeling, including, drafting, part creation, creation of drawings with various views of the part, assemblies of the parts, etc.
- b) **Analysis:** Analysis refers to finite element analysis, optimization, and other number crunching engineering analyses. In general, a geometric model is first created and then the model is analyzed for loads, stresses, moment of inertia, and volume, etc.
- c) **Visualization:** Visualization refers to computer graphics, which includes: rendering a model, creation of pie charts, contour plots, shading a model, sizing, animation, etc.



Components of Computer Aided Design

Each of these three areas has been extensively developed in the last 30 years. Several books are written on each of these subjects and courses are available through the academic institutions and the industry.

Most commercial CAD packages (software) consist of only a single component: design or analysis or visualization. However, a few of the vendors have developed an integrated package that includes not only these three areas, but also includes the manufacturing software (CAM). Due to the large storage requirement, integrated packages use either an UNIX workstation or a mainframe platform, and not the popular PC platform. With the improvement in PC computing speed, it's only a matter of time before we see an integrated package run on a PC.

CAD has revolutionized the modern engineering practice; small and large companies use it alike, spending several billion dollars for the initial purchase or lease alone. CAD related jobs are high in demand and the new graduates have advantage over their senior colleagues, as they are more up to date and more productive.

Computer Aided Manufacturing (CAM)

CAM is the next stage of CAD. A part created in CAD can be downloaded and manufactured, without a human hand touching the part. The process is called CAM, and involves CAD, Networking, and NC programming.

Concurrent Engineering

Concurrent Engineering is another powerful CAD concept that has evolved in the 90's. According to this concept, there is an instantaneous communication between the designer, analyst, and manufacturing. Changes made at any of these work centers are immediately passed on to the others and the product is modified without delay. Often, the customer, management, and the marketing people join in and become part of the process. Concurrent engineering saves the valuable time and helps get the product out in the market quicker. Products that use to take years from the date of its concept to the actual production now take only a few weeks, and the final product is better and cost-effective.

Some large organizations have invested in Rapid Prototyping process. In this process, the part is created by a CAD package and downloaded into the rapid prototyping machine; the machine immediately manufactures the part, using a plastic material. This is a good example of concurrent engineering, sometimes referred as Art to Part concept.

CAD/CAM History

The concept of CAD and CAM is relatively new. The usage is linked with the development of computers. The actual application of CAD/CAM in industry, academia and government is only approximately 30 years old. Formal courses in CAD and Finite Element Analysis (FEA) were introduced in 1970's. The major application thrust of CAD came in 1980's, with the availability of PCs and workstations. In its early stage of usage, very few engineering companies could afford the expense of mainframe computers; however, PCs and workstations have evolved into affordable and adequate platform to support comprehensive CAD packages that initially were designed to run on the mainframe platform. A brief history of the evolution of CAD/CAM, according to the decade and the major CAD/CAM developments, is outlined below.

1960's

- Development in Interactive computer graphics research
- Sketchpad system developed by Ivan Sutherland in 1962
- CAD term coined
- First major commercial CAD/CAM software available: CADAM by Lockheed, in 1965
- Bell Telephone's - Graphics 1 remote display system developed

1970's

- Application of CAM in government, industry and academia
- National organization formed
- Beginning of usage of computer graphics
- Turnkey system available for drafting
- Wireframe and surface modeling software became available
- Mass property calculation and FEA software became available
- NC tape generating, verification, and integrated circuit software became available

1980's

- CAD/CAM used for engineering research and development
- New CAD/CAM theories and algorithms developed
- Integration of CAD/CAM
- Solid modeling software became available
- Use of PCs and workstation began

1990's

- Concept of concurrent engineering developed
- Increased use of CAD/CAM on PCs and workstations
- Improvements in hardware and software

CAD Hardware

There are basically two types of devices that constitute CAD hardware: a) Input devices, and b) Output devices. A brief description follows.

Input Devices

These are the devices that we use for communicating with computer, and providing our input in the form of text and graphics. The text input is mainly provided through keyboard. For graphic input, there are several devices available and used according to the work environment. A brief description of these devices is given here.

Mouse: This is a potentiometric device, which contains several variable resistors that send signals to the computer. The functions of a mouse include locating a point on the screen, sketching, dragging an object, entering values, accepting a software command, etc. Joystick and trackballs are analogous to a mouse device, and operate on the same principle.

Digitizers: Digitizers are used to trace a sketch or other 2-D entities by moving a cursor over a flat surface (which contains the sketch). The position of the cursor provides a feedback to the computer connected with the device. There are electrical wires embedded in orthogonal directions that receive and pass signals between the device and the computer. The device is basically a free moving puck or pen shaped stylus, connected to a tablet.

Light Pens: Lockheed's CADAM software utilized this device to carry out the graphic input. A light pen looks like a pen and contains a photocell, which emits an electronic signal. When the pen is pointed at the monitor screen, it senses light, which is converted to a signal. The signal is sent to the computer, for determination of the exact location of the pen on the monitor screen.

Touch Sensitive Screens: This device is embedded in the monitor screens, usually, in the form of an overlay. The screen senses the physical contact of the user. The new generation of the Laptop computers is a good example of this device.

Other Graphic Input Devices: In addition to the devices described above, some CAD software will accept input via Image Scanners, which can copy a drawing or schematic with a camera and light beam assembly and convert it into a pictorial database.

The devices just described are, in general, independent of the CAD package being used. All commercial CAD software packages contain the device drivers for the most commonly used input devices. The device drivers facilitate a smooth interaction between our input, the software, and the computer. An input device is evaluated on the basis of the following factors:

- Resolution
- Accuracy
- Repeatability
- Linearity

Output Devices

After creating a CAD model, we often need a hard copy, using an output device. Plotters and printers are used for this purpose. A plotter is often used to produce large size drawings and assemblies, where as, a laser jet printer is adequate to provide a 3-D view of a model. Most CAD software require a plotter for producing a shaded or a rendered view.

CAD Software

CAD software are written in FORTRAN and C languages. FORTRAN provides the number crunching, where as, C language provides the visual images. Early CAD packages were turnkey systems, i.e., the CAD packages were sold as an integrated software and hardware package, with no flexibility for using second vendor hardware (1970s and 80s). These systems were based on 16-bit word, and were incapable of networking. The modern CAD software utilizes the open architecture system, i.e., software vendors do not design and manufacture their own hardware. Third party software can be used to augment the basic CAD package. Most popular CAD package will facilitate integration of the Finite Element Analysis and other CAD software from more than one vendor. For example, IDEAS preprocessor can work with almost all the FEA packages for pre and post analyses.

Networking is an important consideration in applications of CAD software. A model created by one engineer must be readily accessible to others in an organization, which is linked by a LAN or other means. The designer, analyst, management, marketing, vendor, and others generally share a model. This is the concurrent engineering in action, mentioned earlier.

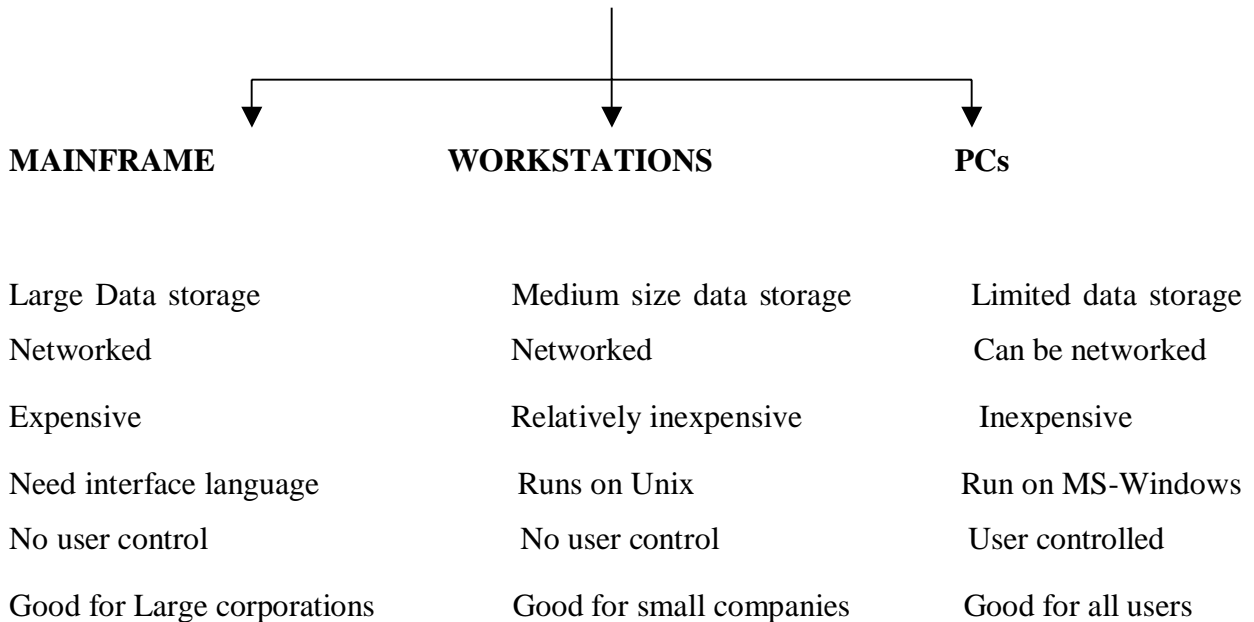
CAD Platform

In general, we can run CAD software on three different CAD platforms: Mainframe, Workstation, and PC. When the CAD programs first became available, they could only be run on a mainframe computer. However, as the PCs have become faster and cheaper, almost all the CAD vendors have introduced a version of their CAD software that will effectively run on a Pentium or higher computer. Currently, the most popular platforms are PCs and Workstations. Popularity of Workstations stems from their ability to network easily with other computers, and also, due to their large memory storage capability.

However, PC platform is still the most preferred medium for most engineers. Increasing popularity of the PC platform can be attributed to several factors, including, total user control, the speed, capability of storing large memory, ease of hardware upgrading and maintenance,

and the overall reasonable cost.

CAD PLATFORMS



CAD Evaluation Criteria

In the current CAD market, ProE and AutoCAD are arguably the most dominating CAD software. AutoCAD is basically a 2-D program, with some capability to create 3-D models, where as, ProE is a truly 3-D CAD package. Besides these software, there are several other CAD software, listed in the previous section (Sec 1.3), that have sales exceeding \$100 millions. No one CAD package is suitable for all the CAD users in the world. The product we are designing dictates the type of CAD package we need. A good CAD package includes good software, as well as, a compatible hardware. Following is a brief description of the general criteria for evaluating a CAD package.

Hardware: Most desirable features in a good hardware are:

- Open architecture
- High speed, large storage
- Compact size
- Inexpensive components
- Inexpensive upgrading

Software: In general, the most comprehensive software are written to satisfy almost all the modeling needs of a modeler, consequently, the software tend to be very complex and hard to learn. To create a simple model, we go through several unnecessary steps, and lack the intuitiveness of a simple, straightforward program. ProE is a good example, where we have to go through several layers of menus to create a simple solid. On the other hand, if we were to use a simpler CAD program, the same solid can be created by only a few simple commands. There are several other factors that we should consider when evaluating software. Following is a brief description of these factors.

- **Operating System:** Unix or Windows/NT. PCs in general use Microsoft Windows, whereas, operating system for Workstations is Unix. For a large organization, Workstations are preferable.
- **User Interface:** Most popular CAD software have menu driven commands, which is preferable to the old system of non-menu driven, where user interface was completely by responding to software commands. The most popular CAD programs work with menu driven interface, with some input/action required through command prompts.
- **Documentation and Support:** Learning a software can be very difficult if the software lacks good documentation. Documentation usually comes in the form of a user's manual, a tutorial book, commands manual, and on-line help. The recent trend is to provide access to the above-mentioned documentation through the Internet, or provide the manuals on a CD ROM. Some CAD vendors provide additional technical support help through phone – ProE is a very good example of this type of support.
- **Maintenance:** Cost of the hardware and software upgrades can significantly impact the small and medium size companies' decision to choose one software over the others. Most CAD vendors go through an upgrade, on the average, every two years. Usually, hardware upgrade is not as frequent.

- **Modeling Capabilities:** In, general, a CAD software can be classified as either a 2-D or a 3-D program. If we were basically involved in 2-D drawings, any well established 2-D software, similar to AutoCAD would suffice our needs. On the other hand, if we need to create 3-D models and assemblies, we will be better off with a 3- D molder – ProE, SOLIDWORKS, etc.
- **Ease of Modeling:** As a rule-of-thumb, a general, all-purpose type CAD software is much more complex and difficult to learn than a special purpose CAD package.
- **Interface with other CAD Packages and Data Transferability:** A CAD package is used to create models that will be used for analysis, manufacturing, or some other applications. Therefore, a CAD software should be capable of transferring and accepting files from other CAD or CAM programs, without this provision, the CAD program has only a very limited use.
- **Design Documentation:** Besides creating a model, the software should be capable of creating drawings, assemblies, dimensioning, various views (isometric, orthogonal, etc.), labels and attributes, etc.

Mechanical Engineering Applications of CAD

Following is a brief description of the applications of CAD in mechanical engineering.

- **Two Dimensional Drafting:** This is the most common use of a CAD package. 2-D drawings are used for manufacturing a product.
- **Report Generating:** To generate reports and bill of materials. Spreadsheets and word-processors can be linked to provide a report writing facility.
- **3-D Modeling:** To create the wireframe, surface and solid models. The 3-D models are for concept verification, manufacturing, FEA, etc.
- **Finite Element Analysis:** FEA package is used for pre-processing, analysis, and post-analysis of structures. For this application, a CAD package contains both the modeling and analysis modules.
- **Manufacturing:** manufacturing software is usually called CAM, and contains CAD software as one of the components. CAM software provides capabilities of carrying out 2 and 3-axes machining.

Module2

- *Syllabus: Transformation of points and line, 2-D rotation, reflection, scaling and combined transformation, homogeneous coordinates, 3-D scaling. Shearing, rotation, reflection and translation, combined transformations, orthographic and perspective projections, reconstruction of 3-D objects.*

TWO-DIMENSIONAL TRANSFORMATION

Computer Aided Design consists of three components, namely, Design (Geometric Modeling), Analysis (FEA, etc), and Visualization (Computer Graphics).

Geometric Modeling provides a mathematical description of a geometric object - point, line, conic section, surface, or a solid. Visualization deals with creation of visual effects, e.g., creation of pie charts, contour plots, shading, animation, etc. Computer graphics provides visual displays and manipulations of objects, e.g., transformation, editing, printing, etc. Fortran and visual C languages are used to effect these operations.

Transformation is the backbone of computer graphics, enabling us to manipulate the shape, size, and location of the object. It can be used to effect the following changes in a geometric object:

- Change the location
- Change the Shape
- Change the size
- Rotate
- Copy
- Generate a surface from a line
- Generate a solid from a surface
- Animate the object

Fa Two-Dimensional Transformation

Geometric transformations have numerous applications in geometric modeling, e.g., manipulation of size, shape, and location of an object. In CAD, transformation is also used to generate surfaces and solids by sweeping curves and surfaces, respectively. The term 'sweeping' refers to parametric transformations, which are utilized to generate surfaces and solids. When we

sweep a curve, it is transformed through several positions along or around an axis, generating a surface. The appearance of the generated surface depends on the number of instances of the transformation. A parameter t or s is varied from 0 to 1, with the interval value equal to the fraction of the parameter. For example, to generate 10 instances, the parameter will have a value $t/10$ or $s/10$. To develop an easier understanding of transformations, we will first study the two-dimensional transformations and then extend it to the study of three-dimensional transformations. Until we get to the discussion of surfaces and solids, we will limit our discussion of transformation to only the simple cases of scaling, translation, rotation, and the combinations of these.

Applications of transformations will become apparent when we discuss the surface and solid modeling.

There are two types of transformations:

Modeling Transformation: this transformation alters the coordinate values of the object. Basic operations are scaling, translation, rotation and, combination of one or more of these basic transformations. Examples of these transformations can be easily found in any commercial CAD software. For instance, AutoCAD uses SCALE, MOVE, and ROTATE commands for scaling, translation, and rotation transformations, respectively.

Visual Transformation: In this transformation there is no change in either the geometry or the coordinates of the object. A copy of the object is placed at the desired sight, without changing the coordinate values of the object. In AutoCAD, the ZOOM and PAN commands are good examples of visual transformation.

Basic Modeling Transformations

There are three basic modeling transformations: Scaling, Translation, and Rotation. Other transformations, which are modification or combination of any of the basic transformations, are Shearing, Mirroring, copy, etc.

Let us look at the procedure for carrying out basic transformations, which are based on matrix operation. A transformation can be expressed as

$$[P^*] = [P] [T]$$

where, $[P^*]$ is the new coordinates matrix

[P] is the original coordinates matrix, or points matrix

[T] is the transformation matrix

With the z-terms set to zero, the P matrix can be written as,

$$[P] = \begin{pmatrix} x_1 & y_1 & 0 \\ x_2 & y_2 & 0 \\ x_3 & y_3 & 0 \\ \vdots & \vdots & \vdots \\ x_n & y_n & 0 \end{pmatrix}$$

The size of this matrix depends on the geometry of the object, e.g., a point is defined by a single set of coordinates (x_1, y_1, z_1) , a line is defined by two sets of coordinates (x_1, y_1, z_1) and (x_2, y_2, z_2) , etc. Thus a point matrix will have the size 1×3 , line will be 2×3 , etc.

A transformation matrix is always written as a 4×4 matrix, with a basic shape shown below,

$$[T] = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Values of the elements in the matrix will change according to the type of transformation being used, as we will see shortly. The transformation matrix changes the size, position, and orientation of an object, by mathematically adding, or multiplying its coordinate values. We will now discuss the mathematical procedure for scaling, translation, and rotation transformations.

Scaling

In scaling transformation, the original coordinates of an object are multiplied by the given scale factor. There are two types of scaling transformations: uniform and non-uniform. In the uniform scaling, the coordinate values change uniformly along the x, y, and z coordinates, where as, in non-uniform scaling, the change is not necessarily the same in all the coordinate directions.

Uniform Scaling

For uniform scaling, the scaling transformation matrix is given as

$$[T] = \begin{pmatrix} s & 0 & 0 & 0 \\ 0 & s & 0 & 0 \\ 0 & 0 & s & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Here, s is the scale factor.

Non-Uniform Scaling

Matrix equation of a non-uniform scaling has the form:

$$[T] = \begin{pmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

where, s_x, s_y, s_z are the scale factors for the x, y, and z coordinates of the object.

Homogeneous Coordinates

Before proceeding further, we should review the concept of homogeneous coordinate system. Since the points matrix has three columns for the x, y, and z values, and a transformation matrix is always 4x4 matrix, the two matrices are incompatible for multiplication. A matrix multiplication is compatible only if the number of columns in the first matrix equals the number of row in the second matrix. For this reason, a points matrix is written as,

$$[P] = \begin{pmatrix} x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ x_n & y_n & z_n & 1 \end{pmatrix}$$

Here, we have converted the Cartesian coordinates into homogeneous coordinates by adding a 4th column, with unit value in all rows. When a fourth column, with values of 1 in each row, is added in the points matrix, the matrix multiplication between the [P] and [T] becomes compatible. The values (x₁, y₁, z₁, 1) represent the coordinates of the point (x₁, y₁, z₁), and the coordinates are called as homogeneous coordinates. In homogeneous coordinates, the points (2,3,1), (4,6,2), (6,9,3), (8,12,4), represent the same point (2,3,1), along the plane z = 1, z = 2, z = 3, and z = 4, respectively. In our subsequent discussion on transformation, we will use homogeneous coordinates.

Example 1: If the triangle A(1,1), B(2,1), C(1,3) is scaled by a factor 2, find the new coordinates of the triangle.

Solution: Writing the points matrix in homogeneous coordinates, we have

$$[P] = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 2 & 1 & 0 & 1 \\ 1 & 3 & 0 & 1 \end{pmatrix}$$

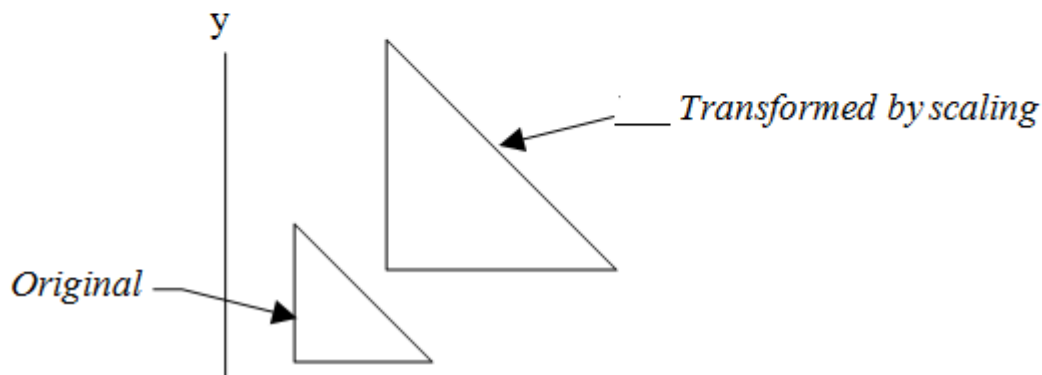
⊕ and the scaling transformation matrix is,

$$[T_s] = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The new points matrix can be evaluated by the equation

$[P^*] = [P] [T]$, and by substitution of the P and T values, we get

$$P^* = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 2 & 1 & 0 & 1 \\ 1 & 3 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 2 & 0 & 1 \\ 4 & 2 & 0 & 1 \\ 2 & 6 & 0 & 1 \end{pmatrix}$$



Translation Transformation

In translation, every point on an object translates exactly the same distance. The effect of a translation transformation is that the original coordinate values increase or decrease by the amount of the translation along the x, y, and z-axes. For example, if line A(2,4), B(5,6) is translated 2 units along the positive x axis and 3 units along the positive y axis, then the new coordinates of the line would be A'(2+2, 4+3), B'(5+2, 6+3) or A'(4,7), B'(7,9).

The transformation matrix has the form:

$$[T_t] = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ x & y & 0 & 1 \end{pmatrix}$$

Rotation

We will first consider rotation about the z-axis, which passes through the origin (0,0,0), since it is the simplest transformation for understanding the rotation transformation. Rotation about an arbitrary axis, other than an axis passing through the origin, requires a combination of three or more transformations, as we will see later.

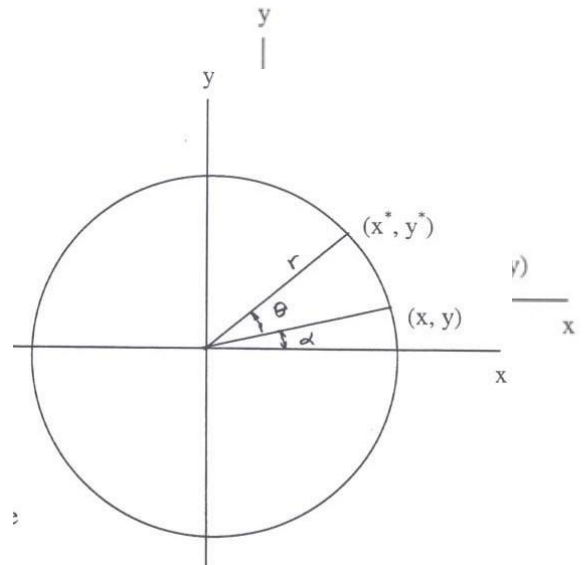
When an object is rotated about the z-axis, all the points on the object rotate in a circular arc, and the center of the arc lies at the origin. Similarly, rotation of an object about an arbitrary axis has the same relationship with the axis, i.e., all the points on the object rotate in a circular arc, and the center of rotation lies at the given point through which the axis is passing.

Derivation of the Rotation Transformation Matrix

Using trigonometric relations, as given below, we can derive the rotation transformation matrix. Let the point P(x, y) be on the circle, located at an angle θ , as shown. If the point P is rotated an additional angle ϕ , the new point will have the coordinates (x*, y*). The angle and the original coordinate relationship is found as follows.

$$\left. \begin{aligned} x &= r \cos \alpha \\ y &= r \sin \alpha \end{aligned} \right\} \text{Original coordinates of point } P.$$

$$\left. \begin{aligned} x^* &= r \cos(\alpha + \theta) \\ y^* &= r \sin(\alpha + \theta) \end{aligned} \right\} \text{The new coordinates.}$$



where, alpha is the angle between the line joining the initial position of the point and the x-axis, and theta is the angle between the original and the new position of the point

Using the trigonometric relations, we get,

$$\begin{aligned} x^* &= r (\cos\alpha \cos\theta - \sin\alpha \sin\theta) = x \cos\theta - y \sin\theta \\ y^* &= r (\cos\alpha \sin\theta + \sin\alpha \cos\theta) = x \sin\theta + y \cos\theta \end{aligned}$$

In matrix form we can write these equations as

$$[x^* \ y^*] = [x \ y] \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \quad (2.8)$$

In general, the points matrix and the transformation matrix given in equation (2.8) are re-written as

Rotation of an Object about an Arbitrary Axis

Rotation of a geometric model about an arbitrary axis, other than any of the coordinate axes, involves several rotational and translation transformations. When we rotate an object about the origin (in 2-D), we in fact rotate it about the z-axis. Every point on the object rotates along a circular path, with the center of rotation at the origin. If we wish to rotate an object about an

arbitrary axis, which is perpendicular to the xy-plane, we will have to first translate the axis to the origin and then rotate the model, and finally, translate so that the axis of rotation is restored to its initial position. If we erroneously use the equation (2.9) directly, to rotate the object about a fixed axis, and skip the translation of this point to the origin, we will in fact end up rotating the object about the z-axis, and not about the fixed axis.

Thus, the rotation of an object about an arbitrary axis, involves three steps: Step 1: Translate the fixed axis so that it coincides with the z-axis

Step 2: Rotate the object about the axis

Step 3: Translate the fixed axis back to the original position.

Combined Transformations

Most applications require the use of more than one basic transformation to achieve desired results. As stated earlier, scaling with an arbitrarily fixed point involves both scaling and translation. And rotation around a given point, other than the origin, involves rotation and translation. We will now consider these combined transformations.

Scaling With an Arbitrary Point

In uniform scaling, all points and their coordinates are scaled by a factor s . Therefore, unless the fixed point is located at $(0, 0)$, it will be moved to a new location with coordinates s -times x and s -times y . To scale an object about a fixed point, the fixed point is first moved to the origin and then the object is scaled. Finally, the object is translated or moved so that the fixed point is restored to its original position.

Mirroring

In modeling operations, one frequently used operation is mirroring an object. Mirroring is a convenient method used for copying an object while preserving its features.

Module3

SURFACES

Introduction

Wire frame models are unable to represent complex surfaces of objects like car, ship, airplane wing, castings etc. A surface model can be used to represent the surface profile of these objects. Also, surface model can be used for calculating mass properties, interference between parts, generating cross-sectioned views, generating finite element mesh, and generating NC tool paths for continuous path machining. Additionally, surface model can be used to fit experimental data, discretized solutions of differential equations, construction of pressure surface, construction of stress distribution etc.

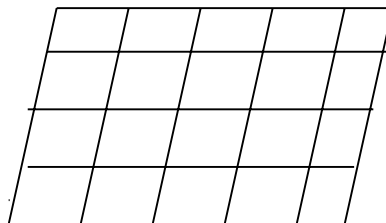
Surface creation on a CAD system usually requires wire frame entities: lines, curves, points, etc. All analytical and synthetic curves can be used to generate surfaces. In order to visualize surfaces on a graphic display, a mesh, say $m \times n$ in size is usually displayed; the mesh size is controlled by the user. Most CAD systems provide options to set the mesh size.

A surface of an object is more complete and less ambiguous representation than its wire frame model; it is an extension of a wire frame model with additional information. A wire frame model can be extracted from a surface model by deleting all surface entities (not the wireframe entities – point, lines, or curves!). Databases of surface models are centralized and associative, manipulation of surface entities in one view is automatically reflected in the other views. Surface models can be shaded and represented with hidden lines.

Types of Surfaces

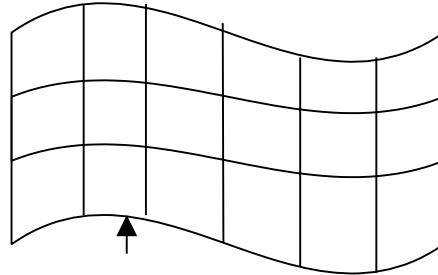
Plane Surface

This is the simplest surface, requires 3 non-coincidental points to define an infinite plane. The plane surface can be used to generate cross sectional views by intersecting a surface or solidmodel with it.



Ruled (lofted) Surface

This is a linear surface. It interpolates linearly between two boundary curves that define the surface. Boundary curves can be any wire frame entity. The surface is ideal to represent surfaces that do not have any twists or kinks.

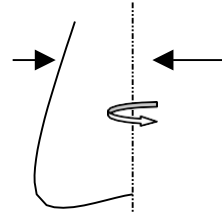


Boundary Curve

Surface of Revolution

This is an axisymmetric surface that can model axisymmetric objects. It is generated by rotating a planar wire frame entity in space about the axis of symmetry of a given angle.

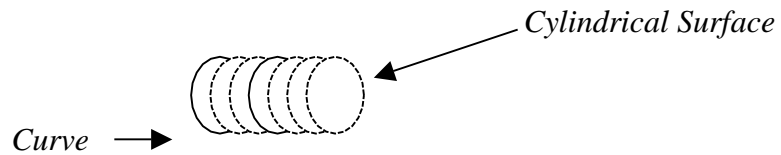
axis of symmetry



Curve

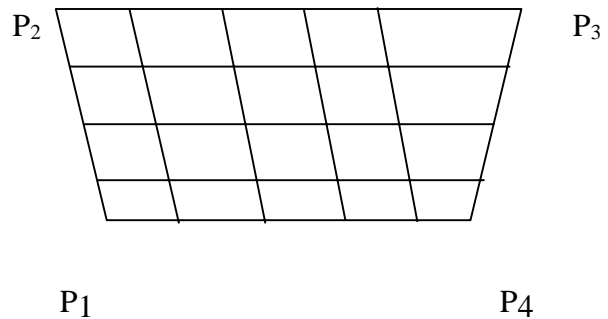
Tabulated Surface

This is a surface generated by translating a planar curve a given distance along a specified direction. The plane of the curve is perpendicular to the axis of the generated cylinder.



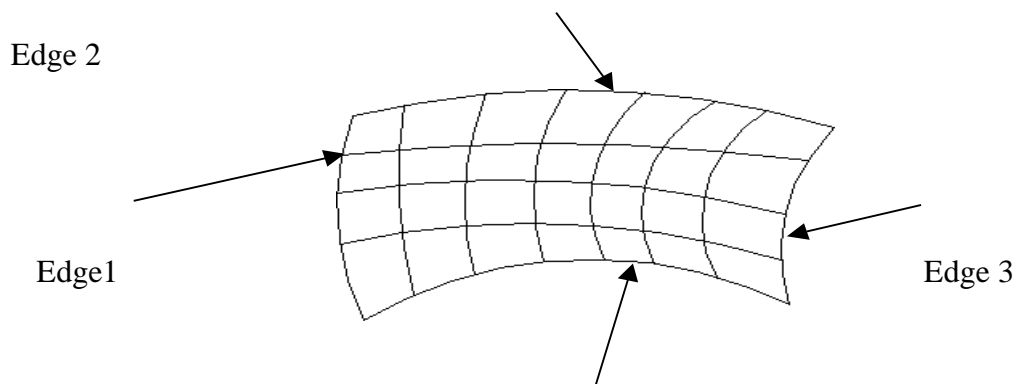
Bi-linear Surface

This 3-D surface is generated by interpolation of 4 endpoints. Bi-linear surfaces are very useful in finite element analysis. A mechanical structure is discretized into elements, which are generated by interpolating 4 node points to form a 2-D solid element.



Coons Patch

Coons patch or surface is generated by the interpolation of 4 edge curves as shown.



Bezier Surface

This is a synthetic surface similar to the Bezier curve and is obtained by transformation of a Bezier curve. It permits twists and kinks in the surface. The surface does not pass through all the data points.

B-Spline Surface

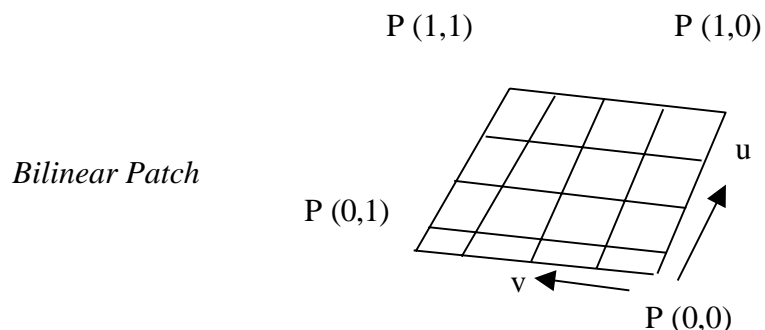
This is a synthetic surface and does not pass through all data points. The surface is capable of giving very smooth contours, and can be reshaped with local controls.

Mathematical derivation of the B-spline surface is beyond the scope of this course. Only limited mathematical consideration will be given here.

Computer generated surfaces play a very important part in manufacturing of engineering products. A surface generated by a CAD program provides a very accurate and smooth surface, which can be generated by NC machines without any room for misinterpretation. Therefore, in manufacturing, computer generated surfaces are preferred. Since surfaces are mathematical models, we can quickly find the centroid, surface area, etc. Another advantage of CAD surfaces is that they can be easily modified.

Interpolated Surfaces – Bilinear Surface

A bilinear surface is obtained by linear interpolation between four points, which may or may not lie in the same plane. The four points appear as vertices or corner points and the parameter values u and v create lines at various intervals to provide the surface visibility, shown in the figure. The parameters u and v are defined as



The interpolated parametric equation of a bilinear surface is given as:

$$P(u,v) = (1-u)(1-v)P(0,0) + u(1-v)P(1,0) + (1-u)vP(0,1) + uvP(1,1)$$

In matrix form, it can be written as

$$P(u,v) = \begin{bmatrix} (1-u)(1-v) & u(1-v) & (1-u)v & uv \end{bmatrix} \begin{matrix} P(0,0) \\ P(1,0) \\ P(0,1) \\ P(1,1) \end{matrix}$$

Node points in FEA

Application of Bilinear Surfaces

Bilinear patches are extensively used in 2-D finite element analysis (FEA). In FEA, an engineering structure is defined by several bilinear surfaces (elements), which are created by joining points on the structure's geometry, called nodes. The nodes are connected to other nodes to create quadrilateral surfaces. Points not lying on the nodes are calculated by interpolation. Thus, the entire structure is completely defined by the nodes and the bilinear surfaces.

Drawbacks of Bilinear Surfaces

Bilinear surfaces have a very limited use, mainly, for FEA. Since only 4 points can be used in the interpolation, the smoothness of the generated surface is limited. Additionally, there is no flexibility to control shapes of the surface, unlike the swept surfaces.

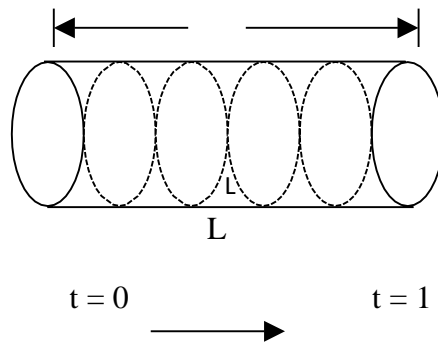
Applications

Coons surface is easy to create, and therefore, many 2-D CAD packages utilize it for generating models. However, it has only a limited application since the surface is inflexible and cannot create very smooth surfaces. It would be very difficult to produce a smooth automobile fender using the Coons surface. Several CAD software, including AutoCAD, use this surface for generating surfaces between 4-bounded edges.

Linearly Swept Surfaces

A swept surface is generated when a curve is parametrically translated or rotated. In CAD, a surface is represented by a series of curves, which are parametrically generated at various instances. For example, a cylindrical surface is generated when a circular arc is translated up to the given dimension using a parameter t , where t varies as, $0 \leq t \leq 1$.

In the figure shown, the cylindrical surface is generated when a circular arc is translated a distance L , with the interim instances at $t = 0.1, 0.2, 0.3, \dots 1$. Here, the parameter t is given 10 values, and therefore, the surface of the cylinder is represented by 10 circular curves. The appearance of the surface improves as the parameter t varies at smaller intervals. Thus, if t is varied with $\Delta t = 0.01$, there will be 100 circular curves representing the surface.



A surface is an extension of a curve. The parametric representation of a curve is given by a single-vector equation of the form:

$$P(t) = [x(t) \quad y(t) \quad z(t)]$$

Here, only one parametric variable or one degree of freedom is needed. Whereas, a surface representation requires two parametric variables, and the equation is given as:

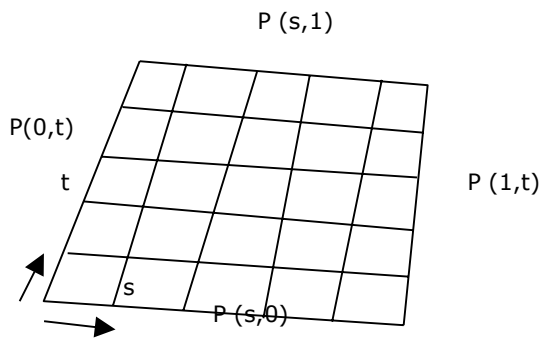
$$Q(s,t) = [x(s,t) \quad y(s,t) \quad z(s,t)]$$

Tracing a point in the s and t directions, as shown in the figure on the next page, generates a surface. One parameter variable is kept constant while varying the other one. A series of curves is created along the s and t directions. For example, constraining the parameters s and t between zero and 1, the set of curves generated along the s direction is,

$$P(0,t), P(0.1,t), P(0.2, t) \dots \dots \dots P(1, t)$$

and the other set of curves along the t direction is,

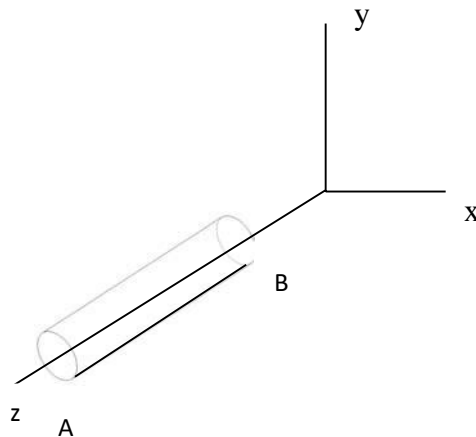
$$P(s,0), P(s,0.1), \dots P(s, 0.9), P(s,1).$$



Thus, creation of a surface requires creation of the multiple curves that constitute it. This concept can be applied to both, the surface that has an analytical formulation (conic sections) and to a free-form surface (Bezier, B-spline).

Revolved Surfaces (Circular Sweep)

Surface of revolution is obtained by rotating a plane-curve around an axis. In the figure shown, line AB is rotated about the z-axis through an

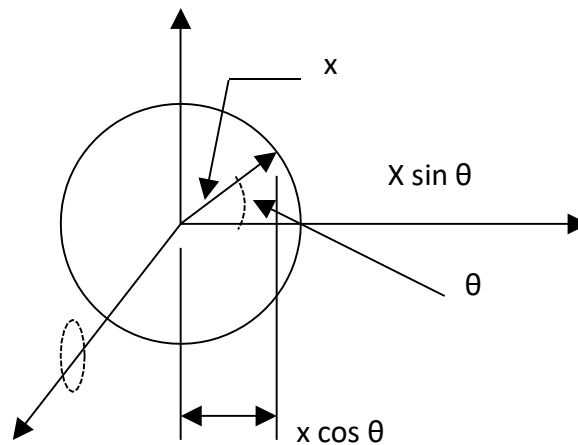


angle of 2π radians, generating a cylinder. A line or curve when revolved can generate all kinds of surfaces, based on the condition of rotation. Any point on the surface is a function of two parameters t and θ . Here, t describes the entity to be rotated and θ represents the angle of rotation. In general, a point on line AB (lying in the xz-plane) is represented by $[x(t), 0, z(t)]$ and, when rotated by θ radians, it becomes $[x(t)\cos\theta, x(t)\sin\theta, z(t)]$.

In general, the point matrix gives a point on the surface of revolution obtained by rotation

around the z-axis,

In matrix form the equation can be written as,



Note: The above rotation matrix is equivalent to the rotational transformation matrix studied earlier, which is, Thus, the generated surface is a rotational transformation of a line (or curve).

Example

Generate the conical surface obtained by rotation of the line segment AB around the z-axis with,

$$A = (1, 0, 1) \text{ and } B = (7, 0, 7).$$

Solution

Line AB can be represented in parametric form as:

$P(t) = [x(t) \ y(t) \ z(t)]$, and the parametric equation of a line is,

$$P(t) = A + (B-A)t$$

based on this equation, the coordinates of a point on the line are given as,

$$x(t) = 1 + (7-1)t = 1 + 6t,$$

$$y(t) = 0$$

$$z(t) = 1 + (7-1)t = 1 + 6t$$

The equation of the surface as given above is,

$$P(t, \theta) = [x(t) \cos \theta \quad x(t) \sin \theta \quad z(t)] \text{ or}$$

$$= [(1+6t) \cos \theta \quad (1+6t) \sin \theta \quad (1+6t)] \quad \text{- equation of the surface}$$

Any point on the surface can be located by substituting t and θ values in the above equation, e.g.: at $t = 0.4$ and $\theta = \pi/2$ radians

$$\begin{aligned} P(0.4, \pi/2) &= [1+6(.4)\cos(\pi/2) \quad 1+6(.4)\sin(\pi/2) \quad 1+6(.4)] \\ &= [0 \quad 3.4 \quad 3.4], \quad \text{which is the point on the surface at } (.4, \pi/2) \end{aligned}$$

Example

Generate a Torus by rotating a circle of radius r and the center at $(a,0,0)$ about the z -axis.

Example:

A cubic Bezier curve is defined by the control points: $P_1 (1,0,2)$, $P_2 (3,0,4)$, $P_3 (2,0,6)$, $P_4 (5,0,7)$. Find the surface of revolution obtained by revolving the curve about the z -axis and calculate the point on the surface at $t = 0.5$, $\theta = \pi/4$ rad.

Solution

The cubic Bezier curve is given by the equation,

$$\begin{aligned} P(t) &= [t][m][v] \\ &= [t^3 \ t^2 \ t \ 1] \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_0 \\ v_1 \\ v_2 \\ v_3 \end{bmatrix} \end{aligned}$$

Substituting the coordinates of the points, we get

$$P(t) = [t^3 \ t^2 \ t \ 1] \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 2 \\ 0 \\ 3 \\ 0 \\ 4 \\ 0 \\ 6 \\ 0 \\ 7 \\ 0 \\ 1 \end{bmatrix}$$

Creating a Surface by Parametric Sweeping

In the examples given above, sweeping a curve parametrically generated the surfaces. In parametric sweeping procedure, a surface is generated through the movement of a line or a curve along or around a defined path. The curve is swept as the sweep parameter is varied from the values of 0 to 1, creating several instances of the curve along the sweep path. In general, the equation of the surface can be given as,

$$Q(t, s) = P(t) T(s)$$

Where, $P(t)$ is the parametric equation of a curve and $T(s)$ is the sweep transformation based on the shape of the path. The sweep transformation can consist of translation, scaling, rotation or a combined transformation. If the path is a straight line, the points along the path on the line can be represented by,

$$\begin{aligned}x(s) &= as \\y(s) &= bs \\z(s) &= cs\end{aligned}$$

and $T(s)$ is given as,

$$T(s) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ as & bs & cs & 1 \end{bmatrix}$$

This is equivalent to a three-dimensional translation of a curve with several traces generated along the path, controlled by how the parameter s is varied.

Example

Consider the Bezier curve defined by the control points $P1 = (0,5,0)$, $P2 = (3,4,0)$, $P3 = (2,0,0)$, and $P4 = (5,0,0)$. Translate the curve five units along the z -axis to generate a swept surface.

Solution

$Q(t,s) = [P(t)] [T]$, substituting the numbers, we get,

$$Q(t, s) = [t^3 \ t^2 \ t \ 1] \begin{bmatrix} -1 & 3 & -3 & 1 & 0 & 5 & 0 & 1 & 1 & 0 & 0 & 0 \\ 3 & -6 & 3 & 0 & 3 & 4 & 0 & 1 & 0 & 1 & 0 & 0 \\ -3 & 3 & 0 & 0 & 2 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 5 & 0 & 0 & 1 & 0 & 0 & 5s & 1 \end{bmatrix}$$

Substituting the value of s and solving the matrices can calculate any point on the surface.

MODULE 4

SOLID MODELING

Application of Solid Models

In mechanical engineering, a solid model is used for the following applications:

1. **Graphics:** generating drawings, surface and solid models
2. **Design:** Mass property calculation, interference analysis, finite element modeling, kinematics and mechanism analysis, animation, etc.
3. **Manufacturing:** Tool path generation and verification, process planning, dimension inspection, tolerance and surface finish.
4. **Component Assembly:** Application to robotics and flexible manufacturing: Assembly planning, vision algorithm, kinematics and dynamics driven by solid models.

Solid Model Representation

There are three different forms in which a solid model can be represented in CAD:

- Wireframe Model
- Surface Model
- Solid Model

Wireframe Models: Joining points and curves creates wireframe models. These models can be ambiguous and unable to provide mass property calculations, hidden surface removal, or generation of shaded images. Wireframe models are mainly used for a quick verification of design ideas.

Surface Models: Surface models are created using points, lines, and planes. A surface model is unable to identify points that do not lie on the surface, and therefore, the moment of inertia, volume, or sections of the model cannot be obtained. A surface model can be shaded for better visibility. Surface models are used for modeling surfaces of engineering components.

Solid models: Solid models are the most preferred form of CAD models. and represent unambiguous

image of a component. A solid model can be used to analyze the moment of inertia, mass, volume, sections of the model, etc.

Solid models are mathematical models of objects in the real world that satisfy specific properties, listed below.

1. **Bounded:** The boundary must limit and contain the interior of the solid.
2. **Homogeneously Three-Dimensional:** No dangling edges or faces be present so that the boundary is always in contact with the interior of the solid.
3. **Finite:** The solid must be finite in size.

Solid Model Creation Scheme

A solid model can be generated by the following schemes.

1. Constructive Solid Geometry (CSG)
2. Boundary Representation (B-Rep)
3. Sweeping

Constructive Solid Geometry Scheme

This scheme is based on the principle that two primitives can be combined to produce a new solid model. This method is also known as 'Building Block' method. The scheme uses the Union, Intersection, and Subtraction techniques to create three-dimensional models, which are based on the Boolean operation. The steps involved in generating a solid model are:

1. Select the primitives from a library
2. Go through the scaling, dimension modification, and any other transformations.
3. Combine the primitives to create the desired solid model.

Since CSG method uses solid primitives, internal details of the object are automatically contained in the model. The model can be sectioned to reveal internal details and can be used for calculating mass, volume, moment of inertia, etc.

New solid models can be created from the primitives or other solid models by the following

operations:

- **Union (U):** Two solids are joined and the common volume of one of the primitives is neglected in the resulting solid.
- **Subtraction or Difference (-):** One solid is subtracted from the other and the resultant solid retains only the uncut portion of the solid.
- **Intersection (∩):** When two solids are combined, the resultant solid represents the common volume of the two solids.

The most common primitive solids found in a CAD program are: Block, Cylinder, Cone, Sphere, Wedge, and Torus.

Boundary Representation (B-Rep) Scheme

This scheme is based on the concept that a physical object is bounded by a set of faces. A solid model is created by combining faces and contains vertices, edges, loops, and bodies. Only the boundary surfaces of the model are stored and the volumetric properties are calculated by the Gauss Divergence theorem, which relates volume integral to surface integrals. This scheme can model a variety of solids depending on the primitive surfaces (planar, curved, or sculptured). There are two types of solid models in this scheme:

1. Polyhedral solids
2. Curved solids

1. Polyhedral Solids: Polyhedral models consist of straight edges, e.g., a non-cylindrical surface: box, wedge, combination of two or more non-cylindrical bodies, etc. Polyhedral solids can have blind or through holes, and two or three-dimensional faces, with no dangling edges. A valid polyhedral abides by the Euler's equation:

$$F - E + V - L = 2 (B - G)$$

Where, F = Face

E = Edge

V = Vertices

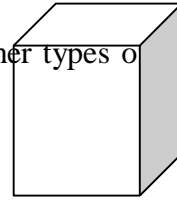
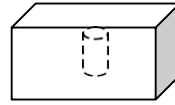
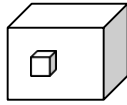
L = Inner Loop B = Bodies

G = Through holes

A simple polyhedral has no holes; each face is bounded by a single set of connected edges (bounded by one loop of edges).

Euler's equation for a simple polyhedral can be reduced to: $F - E + V = 2$

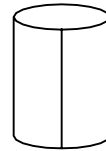
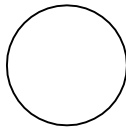
Example: For the box shown, $F = 6$, $E = 12$, and $V = 8$ Examples of other types of polyhedral are shown below.



Polyhedral with two loops

Polyhedral with a blind hole

2. Curved Solids: A curved solid is similar to a polyhedral object but it has curved faces and edges. Spheres and cylinders are examples of curved solids.



Sphere with $F = 1$, $V = 1$, $E = 0$ Cylinder: $F = 3$, $E = 3$, $V = 2$

Primitives: In B-rep, a model is made up of the following primitives:

- **Vertex:** A point in space
- **Edge:** A finite, no-intersecting space curve bounded by two vertices that are not necessarily

distinct.

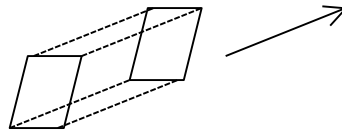
- **Face:** A finite connected, non-self-intersecting, region of a closed oriented surface, bounded by one or more loops.
- **Loop:** An ordered alternating sequence of vertices and edges. A loop defines a non-self-intersecting closed space curve, which may be a boundary of a face.
- **Body:** Entity that has faces, edges and vertices. A minimum body is a point.

B-rep scheme is closely related to the traditional drafting method.

Sweeping Scheme

Sweeping can create a solid model. The method is useful for creating 2 ½ – dimension models. The generated models are axisymmetric and have uniform thickness (i.e., extruded models). There are two types of sweeps: linear and rotational. In linear sweep, a closed 2-D sketch is extruded through the desired length, creating a homogeneous and axisymmetric model, as shown in the figure.

Sweep direction



Linear sweep – Creating a box by sweeping a rectangle

In rotational sweep, a closed sketch is rotated around an axis. The generated model is always axisymmetric.



In addition to the two sweeps described above, a model can also be created by a non-linear sweep. In this type of sweep, a closed sketch is swepted along a non-linear path.

Commercial Modelers

Most commercial software use the sweeping scheme. ProE and Solidworks are good examples of software that utilize sweep technique to generate a 3-D model. In both of these programs, a 2-D sketch is created and extruded to generate a 3-D base-model. The base model is then used to add or modify features. Most engineering components can be created by this technique.

Relatively new software, Ironcad utilizes the CSG technique to create 3-D models. There are pre-build models in the library (catalog) of the software that can be retrieved and modified as needed. The availability of the primitives (basic solid models) accelerates the process of model generation.

All the three software mentioned above are parametric modelers. Another popular software, AutoCAD is capable of generating 3-D models; however, this software is basically a 2-D modeler and lacks the parametric feature. AutoCAD is capable of creating a solid model with CSG, B-rep, and the sweep methods, but limited to only very simple models.

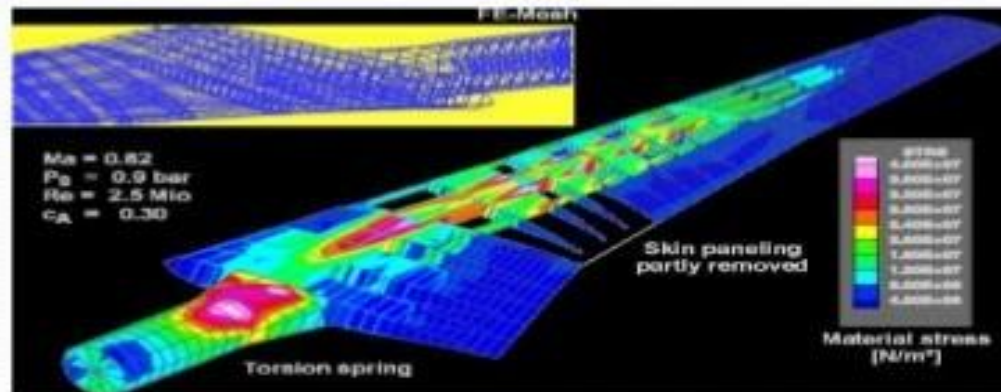
What is Finite Element Analysis (FEA)?

- ❖ The Finite Element Method (FEM) is a numerical technique for finding approximate solutions to boundary value problems for partial differential equations.
- ❖ In simple terms, FEM is a method for dividing up a very complicated problem into small elements that can be solved in relation to each other.
- ❖ Useful for problems with complicated geometries, loadings, and material properties where analytical solutions can not be obtained.

Types Of Engineering Analysis :

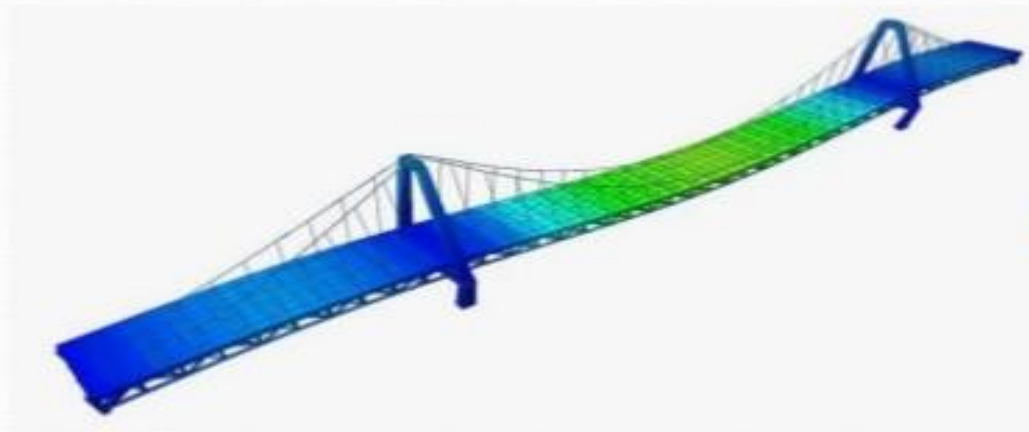
❑ **Structural Analysis :**

Structural Analysis consists of linear and non-linear models. Linear models use simple parameters and assume that the material is not plastically deformed. Non-linear models consist of stressing the material past its elastic capabilities. The stresses in the material then vary with the amount of deformation.



❑ **Vibrational Analysis :**

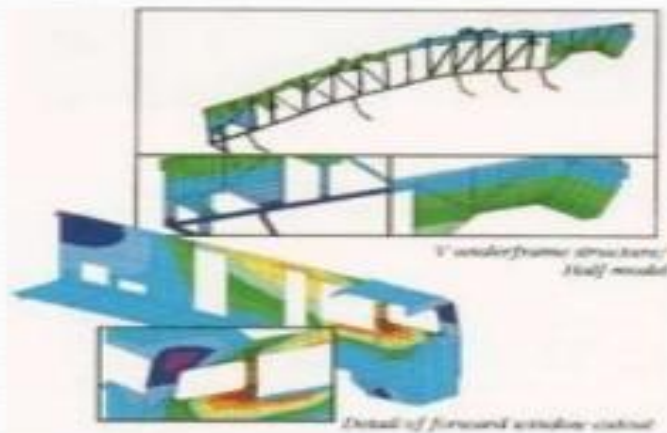
It is used to test a material against random vibrations, shock, and impact. Each of these incidences may act on the natural vibrational frequency of the material which, in turn, may cause resonance and subsequent failure



Analysis of
vibrations in
bridges

❑ **Fatigue Analysis :**

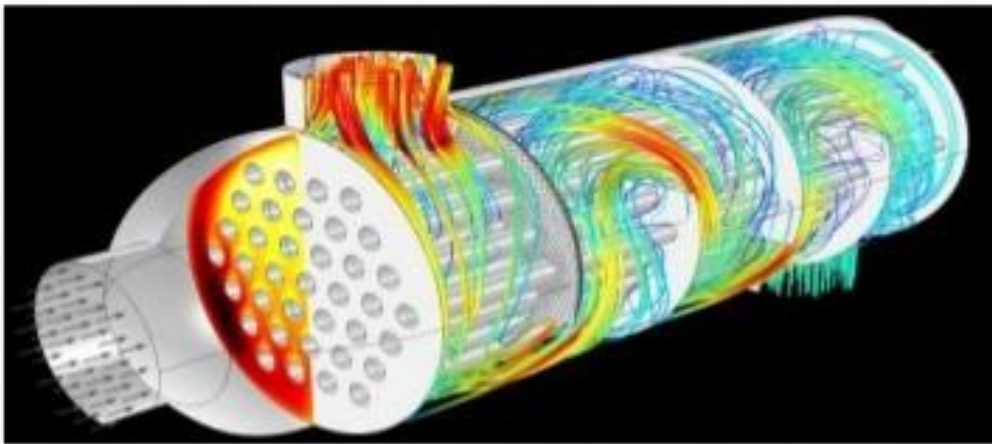
Fatigue analysis helps designers to predict the life of a material or structure by showing the effects of cyclic loading on the specimen. Such analysis can show the areas where crack propagation is most likely to occur. Failure due to fatigue may also show the damage tolerance of the material.



Analysis of fatigue in
frames of a
locomotive

❑ Heat Transfer Analysis :

Heat transfer analysis models the conductivity or thermal fluid dynamics of the material or structure . This may consist of a steady-state or transient transfer. Steady-state transfer refers to constant thermal properties in the material that yield linear heat diffusion.



Thermal Analysis
of Shell & Tube
Heat exchanger

Phases:

Pre-Processing:

Here a finite element mesh is developed to divide the given geometry into subdomains for mathematical analysis and the material properties are applied and also the boundary conditions.

Solution:

In this phase governing matrix equations are derived and the solution for the primary quantities is generated.

Post-Processing:

In the last phase, checking of the validity of the solution generated , examination of the values of primary quantities such as displacement and stresses,errors involved is carried out.

Advantages Of FEA :

- ❖ Can readily handle complex geometry.
- ❖ Can handle complex analysis types like vibration ,heat transfer ,fluids etc.
- ❖ Can handle complex loading:
 - i. Node-based loading (point loads).
 - ii. Element-based loading (pressure, thermal, inertial forces).
 - iii. Time or frequency dependent loading.
- ❖ Can handle complex restraints: Indeterminate structures can be analyzed.
- ❖ Can handle bodies comprised of nonhomogeneous materials: Can handle bodies comprised of non-isotropic materials: Orthotropic & Anisotropic.
- ❖ Special material effects are handled such as temperature dependent properties , plasticity , creep , swelling etc.

Disadvantages Of FEA :

- A specific numerical result is obtained for a specific problem.
- The FEM is applied to an approximation of the mathematical model of a system (the source of so-called inherited errors).
- Experience and judgment are needed in order to construct a good finite element model.
- A powerful computer and reliable FEM software are essential.
- Input and output data may be large and tedious to prepare and interpret.
- Numerical errors such as the limitation of the number of significant digits, rounding –off occur very often.
- Fluid elements with boundaries at infinity can be computed and treated by using boundary element method.

STIFFNESS

Stiffness Matrix is defined as follows:

For an element, a stiffness matrix $\hat{\underline{k}}$ is

a matrix such that $\hat{\underline{f}} = \hat{\underline{k}} \hat{\underline{d}}$, where $\hat{\underline{k}}$ relates local coordinates $(\hat{x}, \hat{y}, \hat{z})$, nodal displacements $\hat{\underline{d}}$ to the local forces $\hat{\underline{f}}$ of a single element.

DERIVATION OF STIFFNESS MATRIX FOR 1D BAR

To derive the stiffness matrix for the linear elastic constant cross sectional area bar element.

The bar is subjected to tensile forces T directed along the local axis of the bar and applied at nodes 1 and 2.

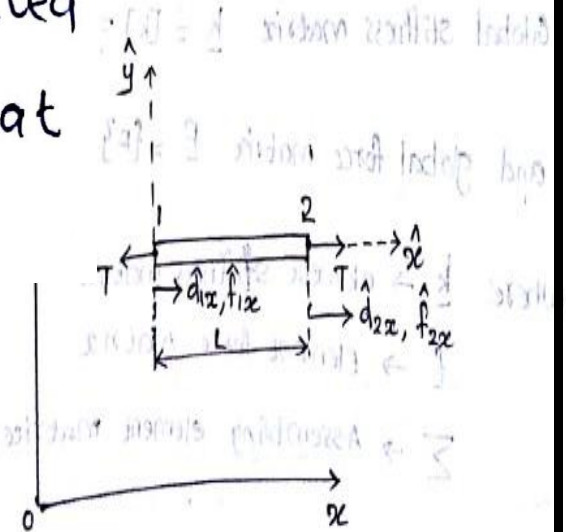


Fig: Bar subjected to tensile forces T .

The following assumptions are used in deriving the bar element stiffness matrix:-

(1) The bar cannot sustain shear force or bending moment, that is $\hat{f}_{1y} = 0, \hat{f}_{2y} = 0,$
 $\hat{m}_1 = 0$ and $\hat{m}_2 = 0.$

(2) Any effect of transverse displacement is ignored.

(3) Hooke's Law applies, ie, axial stress σ_x is related to axial strain ϵ_x by $\sigma_x = E \epsilon_x.$

(4) No intermediate applied loads.

Let

the cross sectional area of bar element = A

modulus of Elasticity = E

Initial length of the element = L

Nodal degrees of freedom = \hat{d}_{1x} and \hat{d}_{2x}
(ie, displacements at nodes 1 & 2)

Nodal forces = f_{1x} and f_{2x}

Steps to derive the stiffness matrix for bar element:

Step 1. Select the element type.
Bar element as shown in figure.

Step 2. Select a displacement function.

Assume a linear displacement variation along \hat{x} axis of the bar element.

$$\hat{u} = a_1 + a_2 \hat{x} \quad \text{--- (1)}$$

In matrix form

$$\hat{u} = \begin{bmatrix} 1 & \hat{x} \end{bmatrix} \begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix} \quad \text{--- (2)}$$

To express \hat{u} as a function of nodal displacements \hat{d}_{1x} and \hat{d}_{2x} .

At node: 1, $\hat{x} = 0$, $\hat{u}(0) = \hat{d}_{1x}$

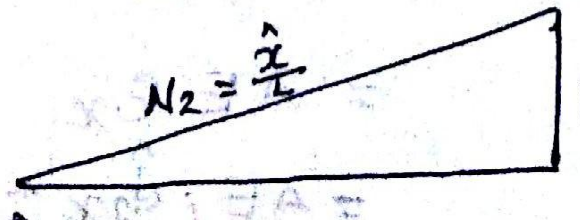
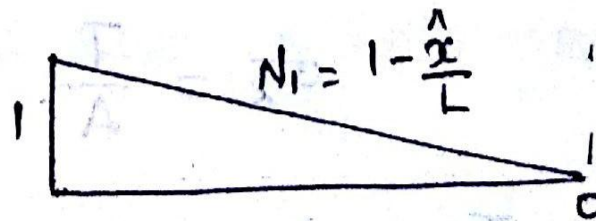
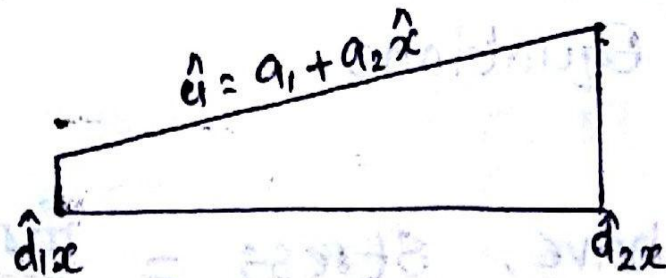
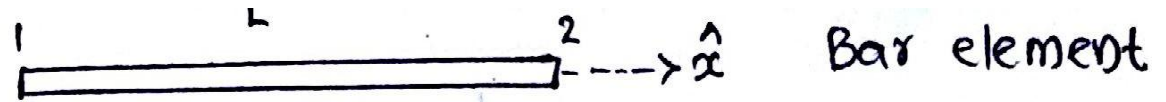
$$\hat{u}(0) = a_1 + a_2 \times 0 = a_1$$

$$\Rightarrow a_1 = \hat{d}_{1x} \quad \text{--- (3)}$$

At node: 2, $\hat{x} = L$, $\hat{u}(L) = \hat{d}_{2x}$

$$\hat{u}(L) = a_1 + a_2 \times L = \hat{d}_{2x}$$

$$\Rightarrow a_2 L = \hat{d}_{2x} - a_1$$



At node 1, ($\hat{x} = 0$)

$$N_1 = 1 - \frac{\hat{x}}{L} = 1 - \frac{0}{L} = \underline{\underline{1}}$$

$$N_2 = \frac{\hat{x}}{L} = \underline{\underline{0}}$$

At node 2, ($\hat{x} = L$)

$$N_1 = 1 - \frac{\hat{x}}{L} = 1 - \frac{L}{L} = \underline{\underline{0}}$$

$$N_2 = \frac{\hat{x}}{L} = \frac{L}{L} = \underline{\underline{1}}$$

Step 3: Define strain-Displacement and stress-strain Relationships.

$$\text{Axial strain, } \epsilon_x = \frac{d\hat{u}}{d\hat{x}} = \frac{\hat{d}_{2x} - \hat{d}_{1x}}{L} \quad (7)$$

$$\text{Axial stress, } \sigma_x = E \epsilon_x \quad (8)$$

Step 4: Derive Element Stiffness matrix and Equations.

We have, $\text{Stress} = \frac{\text{Force}}{\text{Area}}$

$$\sigma_x = \frac{T}{A}$$

$$T = A \sigma_x$$

$$= AE \epsilon_x \quad \text{from (8)}$$

$$= AE \left(\frac{\hat{d}_{2x} - \hat{d}_{1x}}{L} \right) \quad \text{from (7)}$$

Also by the nodal force sign convention,

$$\begin{aligned}\hat{f}_{1x} &= -T \\ &= -AE \frac{\hat{d}_{2x} - \hat{d}_{1x}}{L} \\ &= \frac{AE}{L} (\hat{d}_{1x} - \hat{d}_{2x}) \quad \text{--- (9)}\end{aligned}$$

Similarly

$$\begin{aligned}\hat{f}_{2x} &= T \\ &= \frac{AE}{L} (\hat{d}_{2x} - \hat{d}_{1x}) \quad \text{--- (10)}\end{aligned}$$

Expressing (9) and (10) together in matrix form:

$$\begin{aligned}\begin{Bmatrix} \hat{f}_{1x} \\ \hat{f}_{2x} \end{Bmatrix} &= \begin{bmatrix} \frac{AE}{L} & -\frac{AE}{L} \\ -\frac{AE}{L} & \frac{AE}{L} \end{bmatrix} \begin{Bmatrix} \hat{d}_{1x} \\ \hat{d}_{2x} \end{Bmatrix} \\ &= \frac{AE}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} \hat{d}_{1x} \\ \hat{d}_{2x} \end{Bmatrix} \quad \text{--- (11)}\end{aligned}$$

Step 5: Assemble element equations to obtain

Global Equations.

Global stiffness matrix $\underline{K} = [\underline{k}] = \sum_{e=1}^N \underline{k}^{(e)}$

Global force matrix $\underline{F} = \{F\} = \sum_{e=1}^N \underline{f}^{(e)}$

' \sum ' represents the assemblage of element matrices

Step 6: Solve for Nodal displacements.

By imposing boundary conditions and simultaneously solving a system of equations

$$\underline{F} = \underline{K} \underline{d}$$

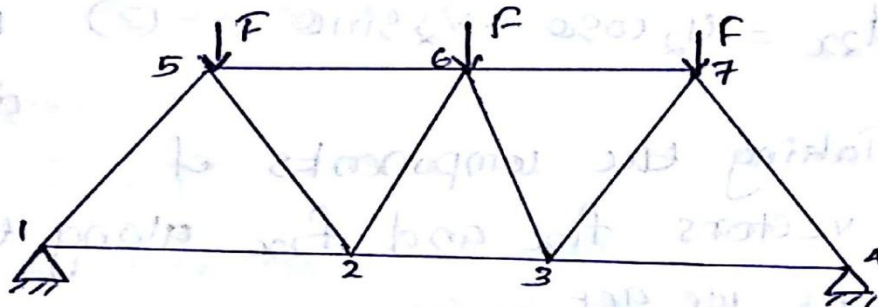
Step 7: Solve for element forces

Finally, determine the strains and stresses in each element by back-substitution of the displacements into equations of ϵ_x and σ_x .

Trusses:

A truss is defined as a structure, made up of several bars, riveted or welded together. The following assumptions are made while finding the forces in a truss.

- 1) All the members are pin jointed.
- 2) The truss is loaded only at joints.
- 3) The self weight of the members are neglected unless stated.



SHAPE FUNCTIONS (or) INTERPOLATION FUNCTIONS

In FEA we discretize the solution region into finite elements. To conduct the analysis we assume a displacement model to approximately indicate the variations of the displacement within the element. The polynomial chosen to interpolate the field variables over the element are called shape functions. Denoted by $[N]$, they establish the relationship between the displacement at any point in the element with the modal displacement of the element.

$$u = N_1u_1 + N_2u_2 + \dots + N_nu_n$$

Two dimensional finite element:-

Two dimensional elements are defined by three or more nodes in a two dimensional plane (*i.e.*, x, y plane). The basic element useful for two dimensional analysis is the triangular element. The simplest two dimensional elements have corner nodes as shown in Fig.3.1. A quadrilateral (special forms of rectangle and parallelogram) element can be obtained by assembling two or four triangular elements, as shown in Fig.3.2. They are often used to model a wide range of Engineering problems.

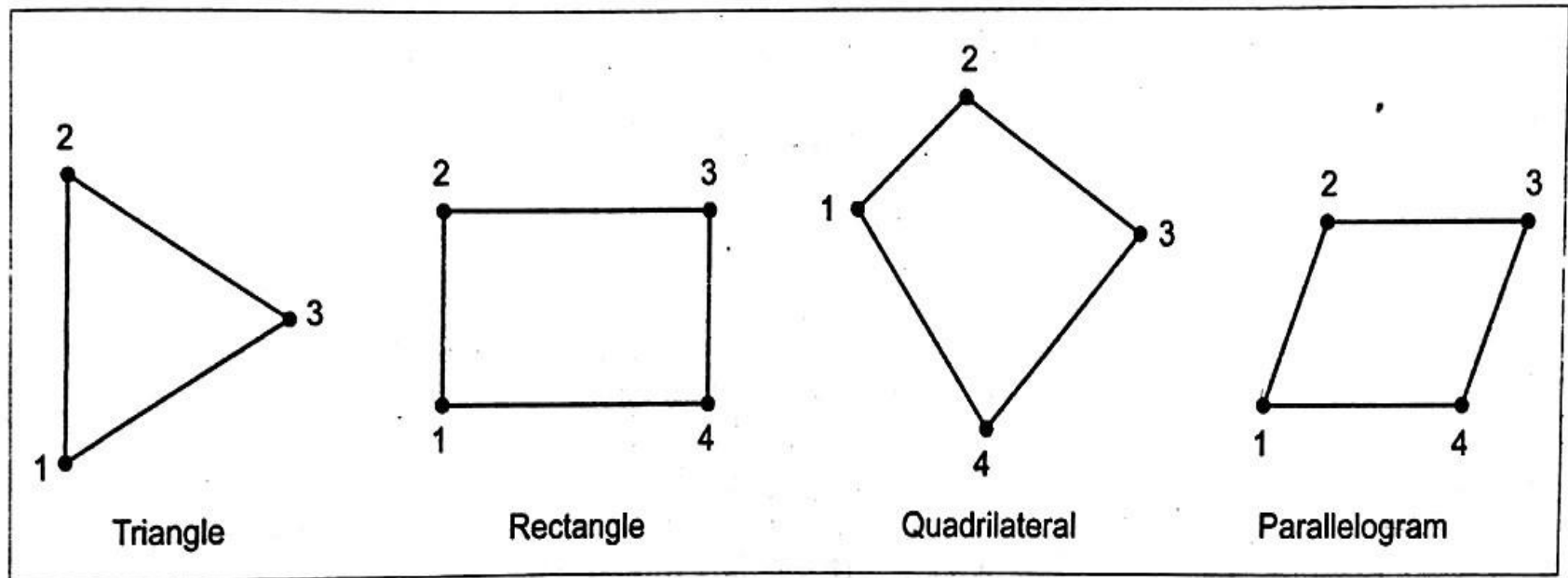


Fig. 3.1. Two dimensional elements

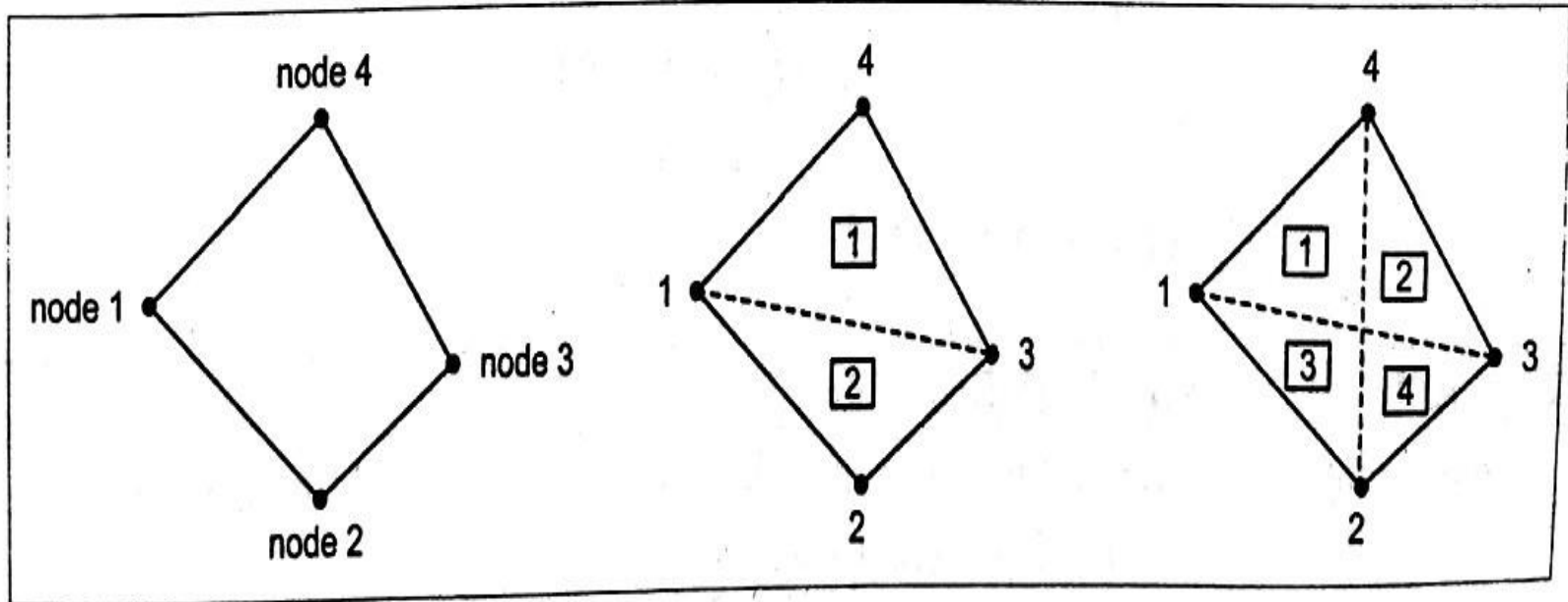


Fig. 3.2. A quadrilateral element as an assemblage of two or four triangular elements

The two dimensional analysis of hydraulic cylinder rod end with plane strain triangular elements is shown in Fig.3.3.

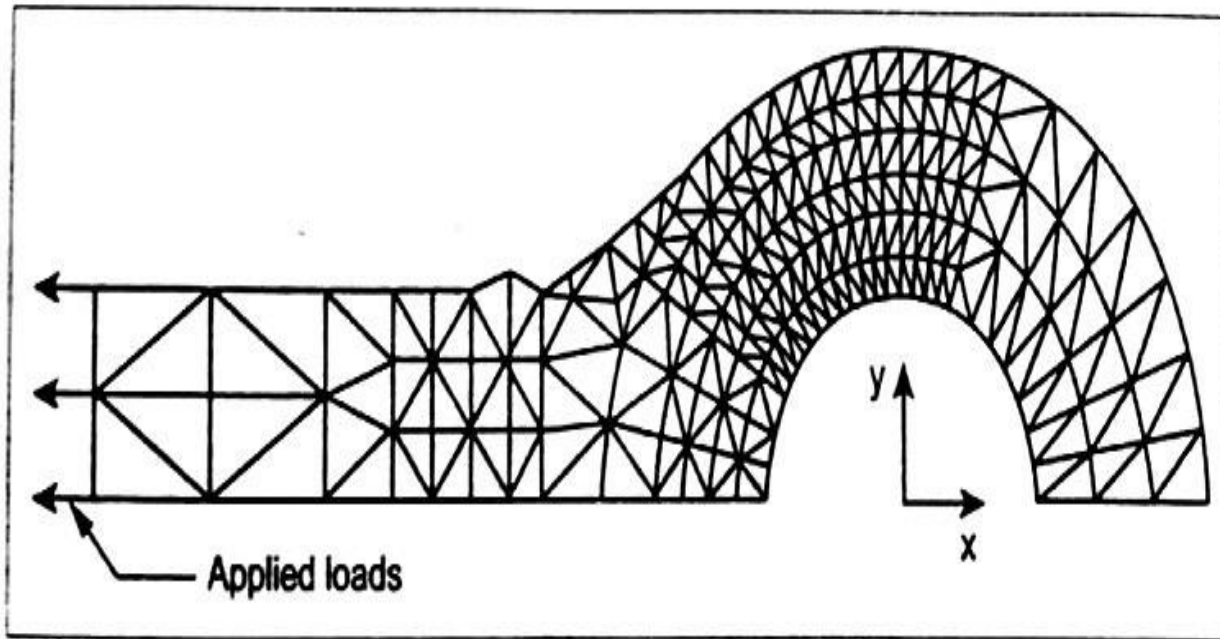


Fig. 3.3. Two dimensional analysis of hydraulic cylinder rod end

The two dimensional finite element formulation follows the same steps which is used in the one dimensional problems. The displacements and distributed body force values are functions of the position indicated by (x, y)

The displacement vector u is given by, $u = \begin{Bmatrix} u \\ v \end{Bmatrix}$

where u and v are the x and y components of u respectively.

The stresses and strains are given as,

$$\sigma = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}$$

$$e = \begin{Bmatrix} e_x \\ e_y \\ \gamma_{xy} \end{Bmatrix}$$

where, $\sigma \rightarrow$ Normal stress

$\tau \rightarrow$ Shear stress

$e \rightarrow$ Normal strain.

$\gamma \rightarrow$ Shear strain.

Body force is given by, $F = \begin{Bmatrix} F_x \\ F_y \end{Bmatrix}$

The two dimensional element is extremely important for the following two analysis.

- (i) Plane stress analysis.
- (ii) Plane strain analysis.

(i) *Plane Stress Analysis*

Plane stress is defined to be a state of stress in which the normal stress (σ) and shear stress (τ) directed perpendicular to the plane are assumed to be zero.

Generally, members that are thin (those with a small z dimension compared to the in-plane x and y dimensions) and whose loads act only in the x - y plane can be considered to be under plane stress.

Plates with holes and plates with fillets are coming under plane stress analysis problems.

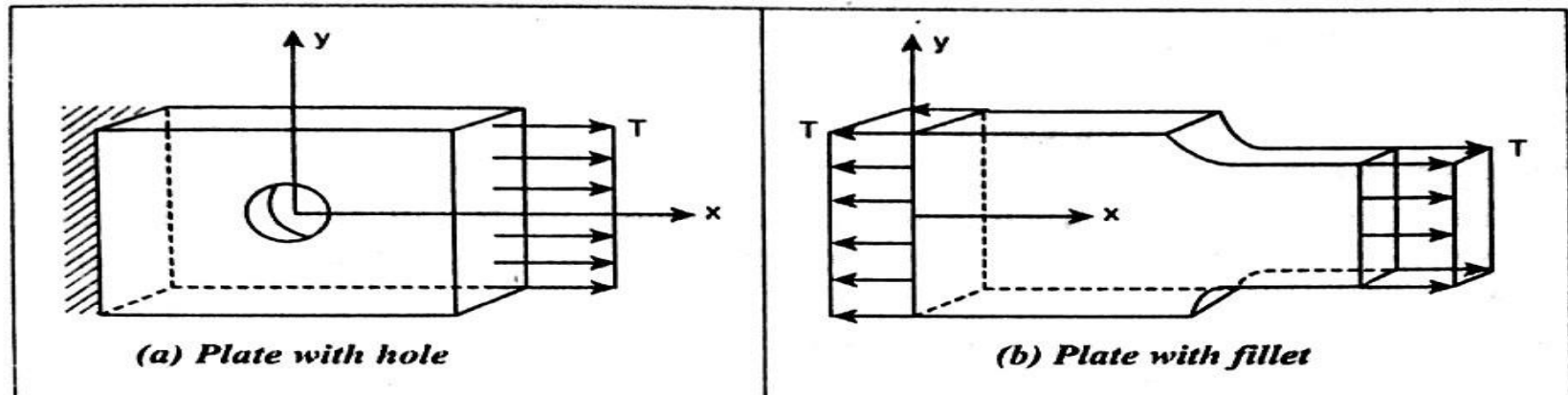


Fig. 3.4. Plane stress problems: (a) plate with hole; (b) plate with fillet

where, $T \rightarrow$ Surface tractions (*i.e.*, pressure acting on the surface edge or face of a member, unit \rightarrow Force/Area \rightarrow N/m^2)

$$\text{Normal stress, } \sigma_z = 0$$

$$\text{Shear stresses } \tau_{xz} \text{ and } \tau_{yz} = 0$$

(ii) Plane strain analysis

Plane strain is defined to be a state of strain in which the strain normal to the xy plane and the shear strains are assumed to be zero.

Dams and pipes subjected to loads that remain constant over their lengths are coming under plane strain analysis problems.

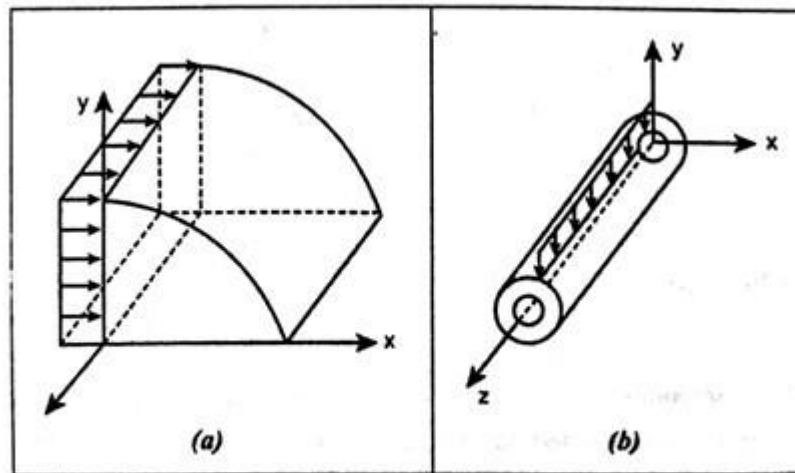


Fig. 3.5. Plane strain problems: (a) dam subjected to horizontal loading; (b) pipe subjected to a vertical load

Here, Normal strain, $e_z = 0$

Shear stresses γ_{xz} and $\gamma_{yz} = 0$

TRIANGULAR

Nodal displacements of the element represents the displacements at points inside an element. Generally, triangular elements are classified into two:

- Constant Strain Triangular Element
- Linear Strain Triangular Element

CONSTANT STRAIN TRIANGULAR ELEMENT (CST)

A three noded triangular element is known as constant strain triangular (CST) element, which is shown in Fig.3.9. It has six unknown displacement degrees of freedom ($u_1, v_1, u_2, v_2, u_3, v_3$). The element is called CST because it has a constant strain throughout it.

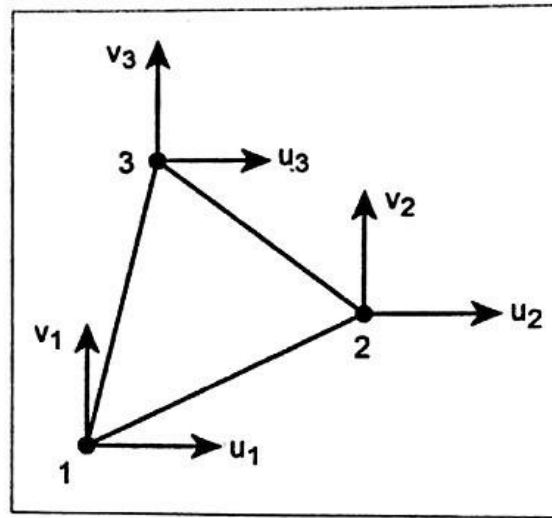


Fig. 3.9. Constant strain triangular element

SHAPE FUNCTION DERIVATION FOR THE CONSTANT TRIANGULAR ELEMENT

We begin this section with the development of the shape function for a basic two dimensional finite element, called constant strain triangular element (CST).

We consider this CST element because its derivation is the simplest among the available two dimensional elements.

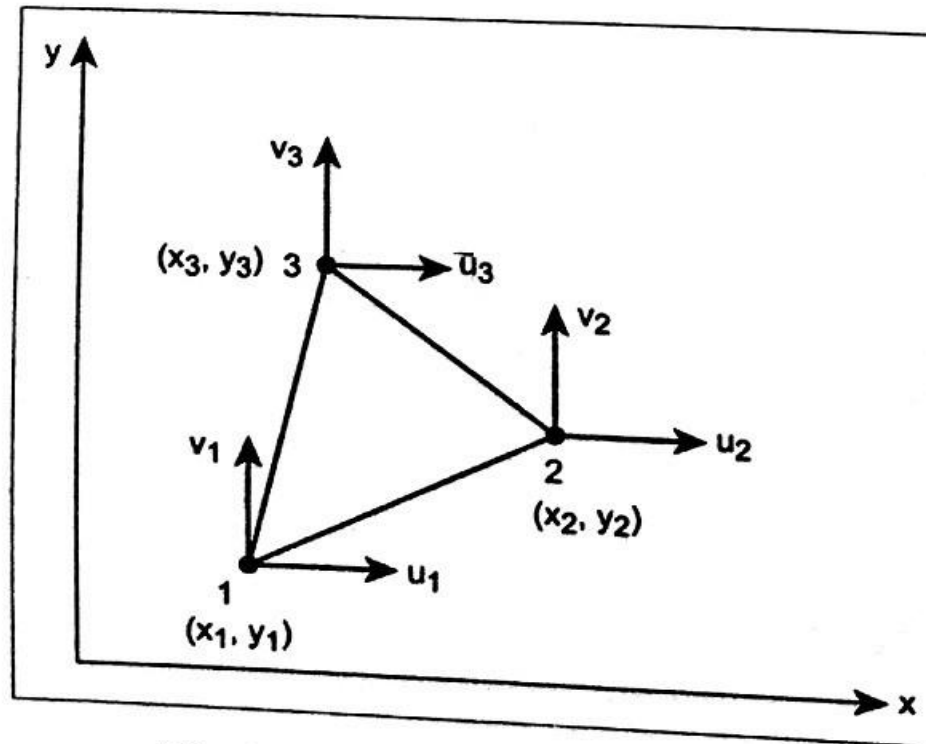


Fig. 3.10. Three noded CST element

Consider a typical CST element with nodes 1, 2 and 3 as shown in Fig.3.10 Let the nodal displacements be u_1, u_2, u_3, v_1, v_2 and v_3 .

$$\text{Displacement } \{ u \} = \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ v_1 \\ v_2 \\ v_3 \end{Bmatrix}$$

Since the CST element has got two degrees of freedom at each node (u, v), the total degrees of freedom is 6. Hence it has 6 generalized coordinates.

$$\text{Let,} \quad u = a_1 + a_2 x + a_3 y \quad \dots (3.1)$$

$$v = a_4 + a_5 x + a_6 y \quad \dots (3.2)$$

where, a_1, a_2, a_3, a_4, a_5 and a_6 are global or generalized co-ordinates.

$$\Rightarrow \quad u_1 = a_1 + a_2 x_1 + a_3 y_1$$

$$u_2 = a_1 + a_2 x_2 + a_3 y_2$$

$$u_3 = a_1 + a_2 x_3 + a_3 y_3$$

Write the above equations in matrix form,

$$\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{bmatrix} \begin{Bmatrix} a_1 \\ a_2 \\ a_3 \end{Bmatrix}$$

$$\text{Let } D = \begin{bmatrix} + & - & + \\ 1 & x_1 & y_1 \\ - & + & - \\ 1 & x_2 & y_2 \\ + & - & + \\ 1 & x_3 & y_3 \end{bmatrix}$$

We know, $D^{-1} = \frac{C^T}{|D|}$

Find the co-factors of matrix D.

$$c_{11} = + \begin{vmatrix} x_2 & y_2 \\ x_3 & y_3 \end{vmatrix} = (x_2 y_3 - x_3 y_2)$$

$$c_{12} = - \begin{vmatrix} 1 & y_2 \\ 1 & y_3 \end{vmatrix} = -(y_3 - y_2) = y_2 - y_3$$

$$c_{13} = + \begin{vmatrix} 1 & x_2 \\ 1 & x_3 \end{vmatrix} = (x_3 - x_2)$$

$$c_{21} = - \begin{vmatrix} x_1 & y_1 \\ x_3 & y_3 \end{vmatrix} = -(x_1 y_3 - x_3 y_1) = x_3 y_1 - x_1 y_3$$

$$c_{22} = + \begin{vmatrix} 1 & y_1 \\ 1 & y_3 \end{vmatrix} = y_3 - y_1$$

$$c_{23} = - \begin{vmatrix} 1 & x_1 \\ 1 & x_3 \end{vmatrix} = -(x_3 - x_1) = x_1 - x_3$$

$$c_{31} = + \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} = x_1 y_2 - x_2 y_1$$

$$c_{32} = - \begin{vmatrix} 1 & y_1 \\ 1 & y_2 \end{vmatrix} = -(y_2 - y_1) = y_1 - y_2$$

$$c_{33} = + \begin{vmatrix} 1 & x_1 \\ 1 & x_2 \end{vmatrix} = x_2 - x_1$$

$$\Rightarrow C = \begin{vmatrix} (x_2 y_3 - x_3 y_2) & (y_2 - y_3) & (x_3 - x_2) \\ (x_3 y_1 - x_1 y_3) & (y_3 - y_1) & (x_1 - x_3) \\ (x_1 y_2 - x_2 y_1) & (y_1 - y_2) & (x_2 - x_1) \end{vmatrix}$$

$$\Rightarrow C^T = \begin{vmatrix} (x_2 y_3 - x_3 y_2) & (x_3 y_1 - x_1 y_3) & (x_1 y_2 - x_2 y_1) \\ (y_2 - y_3) & (y_3 - y_1) & (y_1 - y_2) \\ (x_3 - x_2) & (x_1 - x_3) & (x_2 - x_1) \end{vmatrix} \quad \dots (3.5)$$

We know that,

$$D = \begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix}$$

$$|D| = 1(x_2 y_3 - x_3 y_2) - x_1(y_3 - y_2) + y_1(x_3 - x_2) \quad \dots (3.6)$$

Substitute C^T and D values in equation (3.4),

$$\Rightarrow D^{-1} = \frac{1}{(x_2 y_3 - x_3 y_2) - x_1 (y_3 - y_2) + y_1 (x_3 - x_2)} \times$$

$$\begin{bmatrix} (x_2 y_3 - x_3 y_2) & (x_3 y_1 - x_1 y_3) & (x_1 y_2 - x_2 y_1) \\ y_2 - y_3 & y_3 - y_1 & y_1 - y_2 \\ x_3 - x_2 & x_1 - x_3 & x_2 - x_1 \end{bmatrix}$$

Substitute D^{-1} value in equation (3.3),

$$\begin{Bmatrix} a_1 \\ a_2 \\ a_3 \end{Bmatrix} = \begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{bmatrix}^{-1} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}$$

$$\begin{Bmatrix} a_1 \\ a_2 \\ a_3 \end{Bmatrix} = \frac{1}{(x_2 y_3 - x_3 y_2) - x_1 (y_3 - y_2) + y_1 (x_3 - x_2)} \times$$

$$\begin{bmatrix} (x_2 y_3 - x_3 y_2) & (x_3 y_1 - x_1 y_3) & (x_1 y_2 - x_2 y_1) \\ y_2 - y_3 & y_3 - y_1 & y_1 - y_2 \\ x_3 - x_2 & x_1 - x_3 & x_2 - x_1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} \dots (3.7)$$

The area of the triangle can be expressed as a function of the x, y co-ordinates of the nodes 1, 2 and 3.

$$\Rightarrow A = \frac{1}{2} \begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{bmatrix}$$

$$|A| = \frac{1}{2} [1(x_2 y_3 - x_3 y_2) - x_1(y_3 - y_2) + y_1(x_3 - x_2)]$$

$$\Rightarrow 2A = (x_2 y_3 - x_3 y_2) - x_1(y_3 - y_2) + y_1(x_3 - x_2) \quad \dots (3.8)$$

Substitute $2A$ values in equation (3.7),

$$\Rightarrow \begin{Bmatrix} a_1 \\ a_2 \\ a_3 \end{Bmatrix} = \frac{1}{2A} \begin{bmatrix} (x_2 y_3 - x_3 y_2) & (x_3 y_1 - x_1 y_3) & (x_1 y_2 - x_2 y_1) \\ y_2 - y_3 & y_3 - y_1 & y_1 - y_2 \\ x_3 - x_2 & x_1 - x_3 & x_2 - x_1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} \quad \dots (3.9)$$

$$\Rightarrow \begin{Bmatrix} a_1 \\ a_2 \\ a_3 \end{Bmatrix} = \frac{1}{2A} \begin{bmatrix} p_1 & p_2 & p_3 \\ q_1 & q_2 & q_3 \\ r_1 & r_2 & r_3 \end{bmatrix} \times \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} \quad \dots (3.10)$$

where,

$$p_1 = x_2 y_3 - x_3 y_2; \quad p_2 = x_3 y_1 - x_1 y_3; \quad p_3 = x_1 y_2 - x_2 y_1$$

$$q_1 = y_2 - y_3; \quad q_2 = y_3 - y_1; \quad q_3 = y_1 - y_2$$

$$r_1 = x_3 - x_2; \quad r_2 = x_1 - x_3; \quad r_3 = x_2 - x_1$$

From equation (3.1), we know that,

$$u = a_1 + a_2 x + a_3 y$$

We can write this equation in matrix form.

$$u = [1 \ x \ y] \begin{Bmatrix} a_1 \\ a_2 \\ a_3 \end{Bmatrix}$$

Substitute $\begin{Bmatrix} a_1 \\ a_2 \\ a_3 \end{Bmatrix}$ value, from equation no.(3.10).

$$\Rightarrow u = [1 \ x \ y] \times \frac{1}{2A} \begin{bmatrix} p_1 & p_2 & p_3 \\ q_1 & q_2 & q_3 \\ r_1 & r_2 & r_3 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}$$

$$= \frac{1}{2A} [1 \ x \ y] \begin{bmatrix} p_1 & p_2 & p_3 \\ q_1 & q_2 & q_3 \\ r_1 & r_2 & r_3 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}$$

$$= \frac{1}{2A} [p_1 + q_1 x + r_1 y \quad p_2 + q_2 x + r_2 y \quad p_3 + q_3 x + r_3 y] \times \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}$$

[∵ (1 × 3) × (3 × 3) = 1 × 3]

$$u = \left[\frac{p_1 + q_1 x + r_1 y}{2A} \quad \frac{p_2 + q_2 x + r_2 y}{2A} \quad \frac{p_3 + q_3 x + r_3 y}{2A} \right] \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}$$

The above equation is in the form of

$$u = [N_1 \ N_2 \ N_3] \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} \quad \dots (3.11)$$

$$\text{Similarly, } v = [N_1 \ N_2 \ N_3] \begin{Bmatrix} v_1 \\ v_2 \\ v_3 \end{Bmatrix} \quad \dots (3.12)$$

$$\text{where, Shape function, } N_1 = \frac{p_1 + q_1 x + r_1 y}{2A}$$

$$N_2 = \frac{p_2 + q_2 x + r_2 y}{2A}$$

$$N_3 = \frac{p_3 + q_3 x + r_3 y}{2A}$$

Assembling the equations (3.11) and (3.12) in matrix form,

$$\text{Displacement function, } u = \begin{Bmatrix} u(x, y) \\ v(x, y) \end{Bmatrix} = \begin{bmatrix} N_1 & 0 & N_2 & 0 & N_3 & 0 \\ 0 & N_1 & 0 & N_2 & 0 & N_3 \end{bmatrix} \times \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{Bmatrix}$$

STRAIN-DISPLACEMENT MATRIX [B] FOR CST

Displacement function for CST element is given by,

$$u(x, y) = \begin{Bmatrix} u(x, y) \\ v(x, y) \end{Bmatrix} = \begin{bmatrix} N_1 & 0 & N_2 & 0 & N_3 & 0 \\ 0 & N_1 & 0 & N_2 & 0 & N_3 \end{bmatrix} \times \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{Bmatrix}$$

or we can write

$$u = N_1 u_1 + N_2 u_2 + N_3 u_3$$

$$v = N_1 v_1 + N_2 v_2 + N_3 v_3$$

The strain components for CST element are,

$$\text{Normal strain, } e_x = \frac{\partial u}{\partial x}$$

$$\Rightarrow e_x = \frac{\partial N_1}{\partial x} u_1 + \frac{\partial N_2}{\partial x} u_2 + \frac{\partial N_3}{\partial x} u_3$$

$$\text{Normal strain, } e_y = \frac{\partial v}{\partial y}$$

$$\Rightarrow e_y = \frac{\partial N_1}{\partial y} v_1 + \frac{\partial N_2}{\partial y} v_2 + \frac{\partial N_3}{\partial y} v_3$$

$$\text{Shear strain, } \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

$$\Rightarrow \gamma_{xy} = \frac{\partial N_1}{\partial y} u_1 + \frac{\partial N_2}{\partial y} u_2 + \frac{\partial N_3}{\partial y} u_3 + \frac{\partial N_1}{\partial x} v_1 + \frac{\partial N_2}{\partial x} v_2 + \frac{\partial N_3}{\partial x} v_3$$

Arranging the strains e_x , e_y and γ_{xy} in matrix form,

$$\begin{Bmatrix} e_x \\ e_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} \frac{\partial N_1}{\partial x} & 0 & \frac{\partial N_2}{\partial x} & 0 & \frac{\partial N_3}{\partial x} & 0 \\ 0 & \frac{\partial N_1}{\partial y} & 0 & \frac{\partial N_2}{\partial y} & 0 & \frac{\partial N_3}{\partial y} \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial y} & \frac{\partial N_2}{\partial x} & \frac{\partial N_3}{\partial y} & \frac{\partial N_3}{\partial x} \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{Bmatrix} \quad \dots (3.14)$$

From equation (3.11) or (3.12), we know that,

$$\text{Shape function, } N_1 = \frac{p_1 + q_1 x + r_1 y}{2A}$$

$$N_2 = \frac{p_2 + q_2 x + r_2 y}{2A}$$

$$N_3 = \frac{p_3 + q_3 x + r_3 y}{2A}$$

Partial differentiation,

$$\frac{\partial N_1}{\partial x} = \frac{q_1}{2A}; \quad \frac{\partial N_2}{\partial x} = \frac{q_2}{2A}; \quad \frac{\partial N_3}{\partial x} = \frac{q_3}{2A}$$

$$\frac{\partial N_1}{\partial y} = \frac{r_1}{2A}; \quad \frac{\partial N_2}{\partial y} = \frac{r_2}{2A}; \quad \frac{\partial N_3}{\partial y} = \frac{r_3}{2A}$$

Substitute $\frac{\partial N_1}{\partial x}$, $\frac{\partial N_2}{\partial x}$, $\frac{\partial N_3}{\partial x}$, $\frac{\partial N_1}{\partial y}$, $\frac{\partial N_2}{\partial y}$ and $\frac{\partial N_3}{\partial y}$ values in equation (3.14),

$$(3.14) \Rightarrow \begin{Bmatrix} e_x \\ e_y \\ \gamma_{xy} \end{Bmatrix} = \frac{1}{2A} \begin{bmatrix} q_1 & 0 & q_2 & 0 & q_3 & 0 \\ 0 & r_1 & 0 & r_2 & 0 & r_3 \\ r_1 & q_1 & r_2 & q_2 & r_3 & q_3 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{Bmatrix}$$

The above equation is in the form of $\{e\} = [B] \{u\}$

where, $[B] = \text{Strain-Displacement matrix} = \frac{1}{2A} \begin{bmatrix} q_1 & 0 & q_2 & 0 & q_3 & 0 \\ 0 & r_1 & 0 & r_2 & 0 & r_3 \\ r_1 & q_1 & r_2 & q_2 & r_3 & q_3 \end{bmatrix} \dots (3.15)$

where, $\left. \begin{aligned} q_1 &= y_2 - y_3 \\ q_2 &= y_3 - y_1 \\ q_3 &= y_1 - y_2 \\ r_1 &= x_3 - x_2 \\ r_2 &= x_1 - x_3 \\ r_3 &= x_2 - x_1 \end{aligned} \right\}$

[From equation no. (3.10)]

ISOPARAMETRIC SUPERPARAMETRIC AND ELEMENTS

The finite element method is a powerful technique for analysing engineering problems involving complex and irregular geometrics. However, the two and three dimensional elements (triangle, rectangle, brick) discussed in previous chapters cannot be used efficiently for irregular geometrics.

Consider a continuum shown in Fig.5.1(a) and it is discretized by using triangular elements which is shown in Fig.5.1(b).

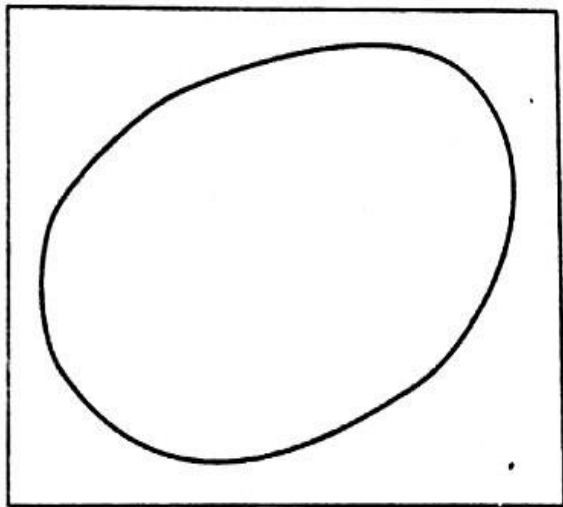
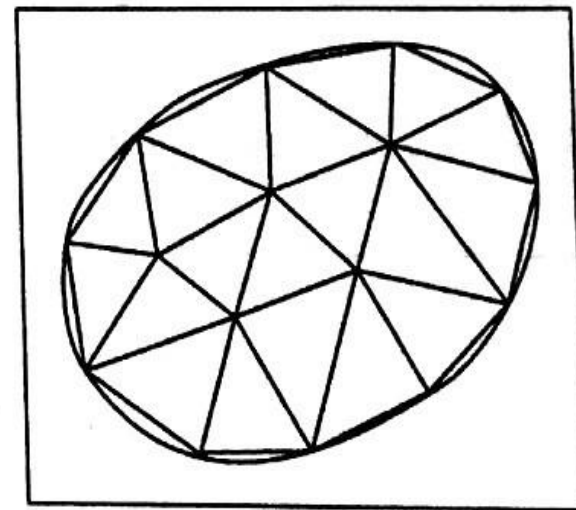


Fig. 5.1. (a) Continuum



***Fig. 5.1. (b) Continuum is discretized
by triangular elements***

It is difficult to represent the curved boundaries by straight edges elements. A large number of elements may be used to obtain reasonable resemblance between original body and the assemblage. In order to overcome this drawback, isoparametric elements are used. *i.e.*, for problems involving curved boundaries, a family of elements known as “isoparametric elements” can be used.

The isoparametric concept was first brought out by *Taig* and latter on generalized by *B.M. Irons* for mapping the curved boundaries. They brought out the concept of mapping for regular triangular, rectangular elements and brick elements from natural co-ordinate system to global cartesian system as shown in Fig.5.2, 5.3 & 5.4.

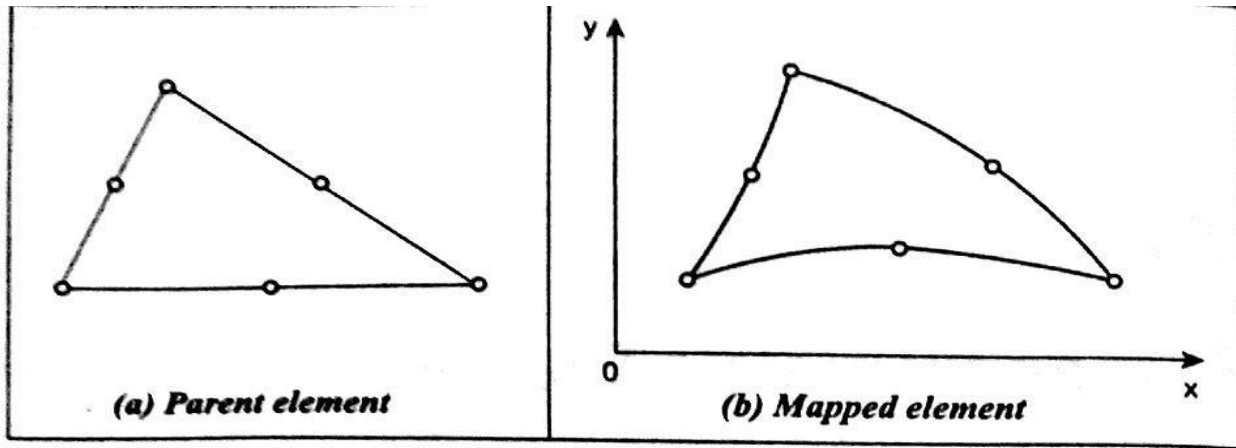


Fig. 5.2. Concept of mapping in isoparametric elements (Triangular element)

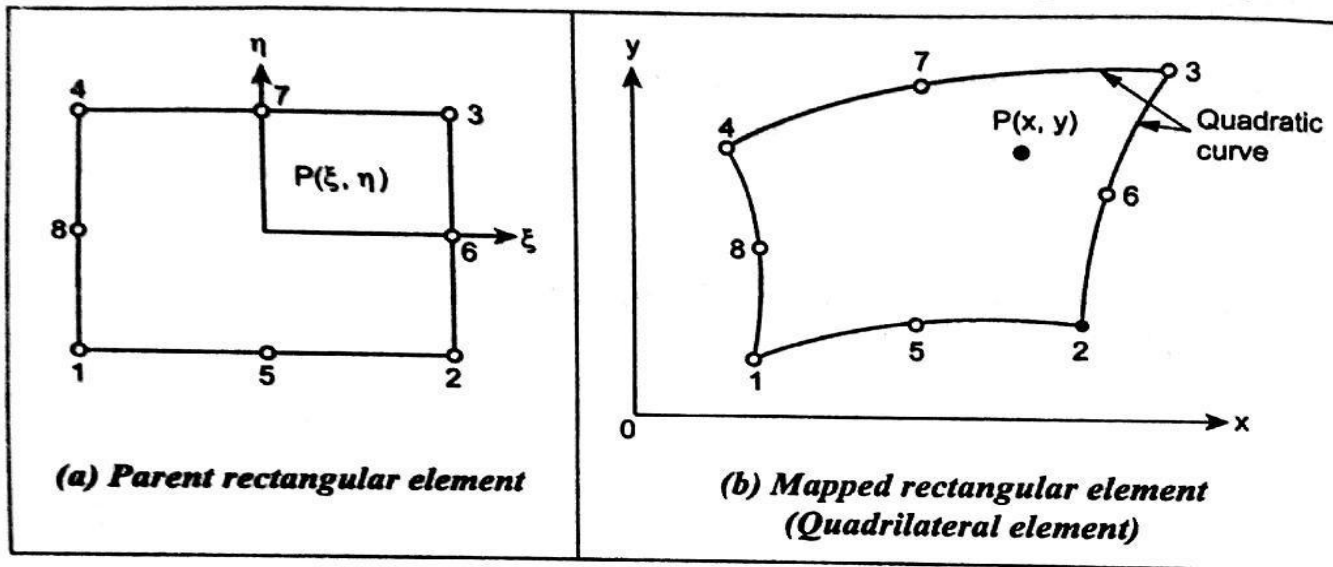


Fig. 5.3.

ISOPARAMETRIC

We know that, shape functions are used for defining the geometry and displacements of the element. Consider a element shown in Fig.5.5.

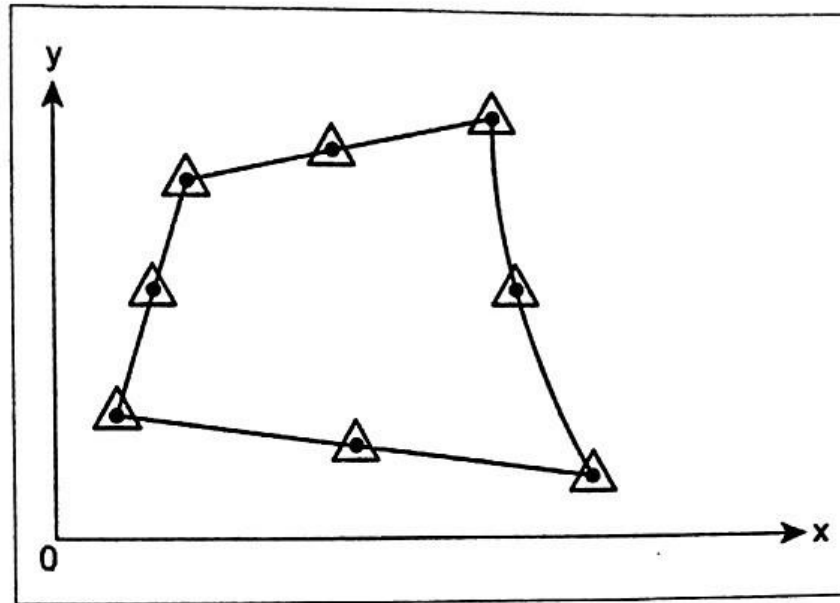


Fig. 5.5.

- Nodes used for defining geometry.
- △ Nodes used for defining displacements.

In this element, all the eight nodes are used in defining geometry as well as displacements.

If the number of nodes used for defining the geometry is same as number of nodes used for defining the displacements, then, it is known as isoparametric element.

SUPERPARAMETRIC

Consider a element shown in Fig.5.6.

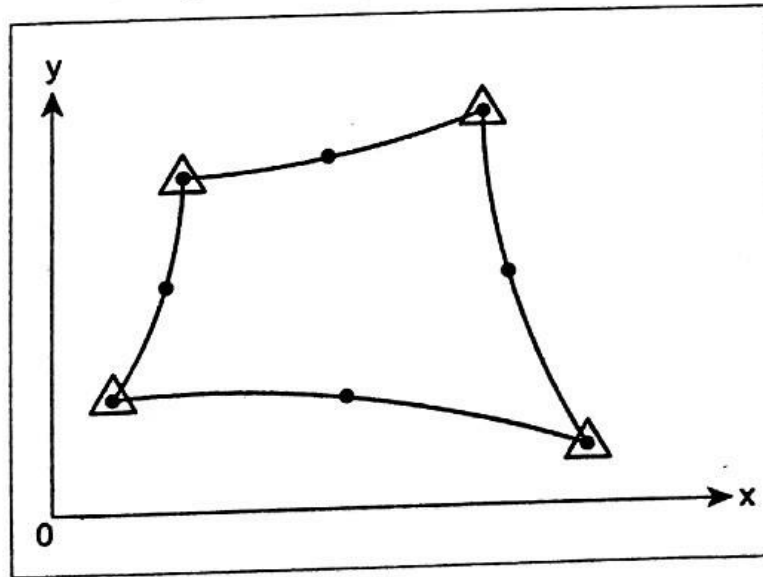


Fig. 5.6.

- Nodes used for defining geometry.
- Δ Nodes used for defining displacements.

In this element, eight nodes are used to define the geometry and four nodes are used to define the displacements.

If the number of nodes used for defining the geometry is more than number of nodes used for defining the displacements, then, it is known as superparametric element

SUBPARAMETRIC

Consider a element shown in Fig.5.7.

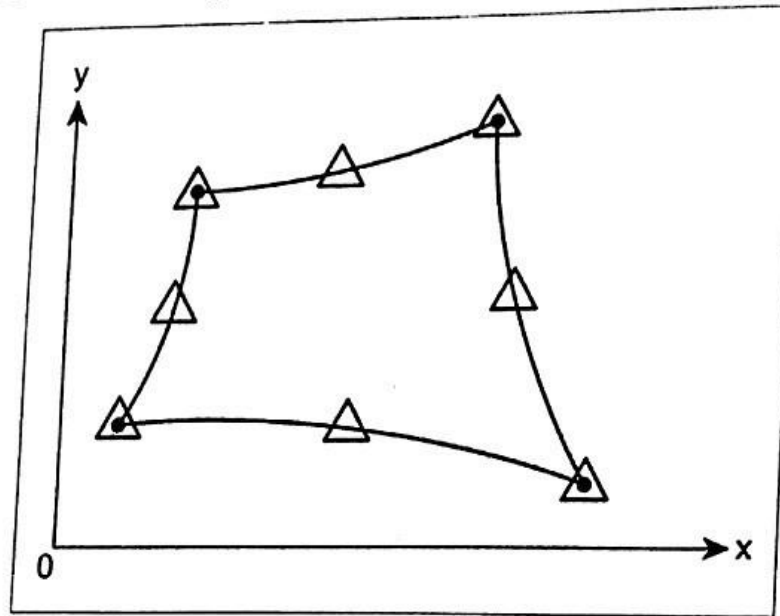


Fig. 5.7.

- Nodes used for defining geometry.
- △ Nodes used for defining displacements.

In this element, four nodes are used to define the geometry and eight nodes are used to define the displacements.

If the number of nodes used for defining the geometry is less than number of nodes used for defining the displacements, then it is known as subparametric element.

NUMERICAL INTEGRATION (GAUSSIAN QUADRATURE)

Numerical integration and Application to Plane Stress Problems

An accurate representation of irregular domains (i.e) domain with curved boundaries, can be accomplished by the use of refined meshes and / or irregularly shaped curvilinear elements.

For example, a non rectangular region cannot be represented accurately using rectangular elements; However, it can be represented by quadrilateral elements.

Since the **interpolation functions** are easily derivable for a rectangular element and it is easier to evaluate integrals over rectangular geometries, we transform the finite element integral statements defined over quadrilaterals to rectangle.

The transformation in complicated expressions in terms of the co-ordinates are used for rectangular element.

Therefore Gaussian Quadrature (numerical integration) is used to evaluate such complicated expressions.

Consider the problem of numerically evaluating a one-dimensional integral of the form.

$$I = \int_{-1}^1 f(x) dx \quad \dots(6.101)$$

The Gaussian quadrature approach for evaluating I is given below.

This method is most useful in finite element work.

The Gauss quadrature is one of the numerical integration methods to calculate the definite integrals. In finite element analysis, Gauss quadrature method is mostly preferred. In this method, the numerical integration is achieved by the following expression,

$$\int_{-1}^1 f(x) dx = \sum_{i=1}^n w_i f(x_i) \quad \dots (5.46)$$

where, w_i is weight function.

$f(x_i)$ is values of the function at pre-determined sampling points.

Function $f(x_i)$ is calculated at several sampling points *i.e.*, $n = 1, 2, 3, \dots$ and each value of $f(x_i)$ is multiplied by weight function w_i . Finally all the terms are added, it gives the value of integration.

Table shows the location of Gauss sampling points $f(x_i)$ and corresponding weight function w_i , for different number of points (n).

Table gives Gauss points for integration from -1 to $+1$,

$$\int_{-1}^1 f(x) dx = \sum_{i=1}^n w_i f(x_i)$$

Table gives the value of W_1 and x_1 for Gauss quadrature formulae of orders $n = 1$ through $n = 4$

Number of points, n	Location, x_1	Corresponding weights, w_1
1	$x_1 = 0.000\dots\dots$	2.000
2	$x_1, x_2 = \pm\sqrt{\frac{1}{3}} = \pm 0.577350269189$	1.000
3	$x_1, x_3 = \pm\sqrt{\frac{3}{5}} = \pm 0.774596669241$ $x_2 = 0.000$	$\frac{5}{9} = 0.555555\dots\dots\dots$ $\frac{8}{9} = 0.888888\dots\dots\dots$
4	$x_1, x_4 = \pm 0.8611363116$ $x_2, x_3 = \pm 0.3399810436$	0.3478548451 0.6521451549